

# Simultaneous Measurements of $O^+$ and $H^+$ Temperatures in the Topside Ionosphere Over Arecibo

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February 4, 2000

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## Abstract

We have measured both proton ( $T_{H^+}$ ) and oxygen ion ( $T_{O^+}$ ) temperatures simultaneously from spectra obtained in the lower topside ionosphere using the Arecibo incoherent scatter radar (ISR). This is the first time that two ion temperatures and three ion compositions have been measured from the same IS spectrum, and, as far as we know, one of the first measurements of both ion temperatures in the ionosphere. We are motivated by the need to evaluate theoretical simulations predicting differences in the two temperatures and also by the need to include all necessary degrees of freedom in our analysis in order to obtain correct values of the other parameters.

First we describe our spectral-domain analysis technique, emphasizing the ability to determine goodness of fit, which allows of the need for and effect of including a parameter to be numerically evaluated, and visual monitoring, which gives clues as to how to proceed. We show that common  $T_{H^+}$  and  $T_{O^+}$  allow insufficient freedom, and that separate temperatures are sufficient. We consider and dismiss some alternative explanations.

The errors can be as low as 100 K on both ion temperatures, but the altitude range of the measurement is somewhat restricted. A small negative bias in  $[He^+]$  parameter is removed when the second temperature is allowed to be free while the  $H^+$  fraction is not significantly affected. Finally we also show that the measurement cannot be made at altitudes where little  $O^+$  is present.

The difference between  $T_{H^+}$  and  $T_{O^+}$  is zero (or at least very small) during the night, and rises to several hundred degrees at about 500 km shortly after sunrise. During the middle of the day the temperature difference declines (presumably due to higher densities) and then rises again before sunset.

## 1 Introduction

The earliest experimental studies of the topside ionosphere were done in the early 1960's using spacecraft and incoherent scatter radars, both emerging technologies at

the time. These studies focused on the concentrations of the major species in the ionospheric plasma. When the first plasma temperature measurements were made, it was quickly determined that above 300 km or so during the day the temperature of the ions was higher than that of the ambient neutral atmosphere and that the electron temperature was higher than the ion temperature. This led to the hypothesis by [Dalgarno and Walker, 1966] that since the efficiency of the heating and cooling mechanisms depended on the individual ions mass, the different ion species could have different temperatures.

The incoherent scatter radar technique is a well-established tool for research in the upper atmospheric physics field and is the most powerful ground-based technique for probing the ionosphere. The Arecibo Observatory is located near Arecibo, P.R. (18.35° N, 66.75° W), which is at a latitude of about 30° N geomagnetic (or 46.7deg dip latitude). It is the very large aperture of the Arecibo radar that makes it so useful for topside work.

The Arecibo radar has been used to measure composition and ion temperature in the topside ionosphere for many years. [Moorcroft, 1969] studied nighttime composition and temperature of the topside ionosphere over Arecibo and pointed out that the interpretation was complicated by the mixture of ions; thus he considered only nighttime data where he could assume  $T_e = T_i$ . In one of the first papers describing electron temperature measurements at Arecibo [Carlson and Sayers, 1970] studied the discrepancy between ISR and Langmuir probe measurements of  $T_e$ . In theory it was possible to measure the temperatures of all the species independently, in practice this was limited to one temperature presumably due to limitations in the fitting techniques. [Hagen and Hsu, 1974] pointed out that the helium variable was strongly correlated with the electron-to-ion temperature ratio. In their study they measured the topside composition during the night by assuming  $T_e/T_i = 1$ , where  $T_i$  is the single temperature applying to all ionic species, and neglected helium to measure  $T_e/T_i$  during the day. As far as we know, [Munninghoff and Bowhill, 1981] made the only previous measurement of separate  $T_{H^+}$  and  $T_{O^+}$ . They show theoretical examples of how the ACF depends on  $T_{H^+}/T_{O^+}$  and two estimated  $T_{H^+}/T_{O^+}$  profiles using data obtained in 1972. To obtain these estimates they neglected  $He^+$  and compared the measured ACF's with a library of theoretical ones. They found an altitude region in the lower topside, between 300 and 700 km, where the protons had 1.3 to 1.5 higher temperatures than the oxygen ions during the day. Their results are similar to ours, but improvements in sensitivity and processing permit allow current results to have much better accuracy and resolution. More recently, [Diloy et al., 1996] did a theoretical study on the possibility of measuring  $H^+$  and  $O^+$  temperatures using the EISCAT ISR. They correctly concluded that it would be difficult to this measurement at EISCAT, for it is the very high sensitivity of the Arecibo radar that makes these measurements relatively easy in the lower topside.

Many of the theoretical models currently in use solve the coupled continuity, momentum and energy equations. In theory they have the capabilities to simulate the different temperatures, in practice this is rarely done. One that does is described in [Bailey and Sellek, 1990]. They show the variation of  $T_{O^+}$ ,  $T_{He^+}$ ,  $T_{H^+}$  and  $T_e$  for  $L = 3$ . In another more recent simulation [Guter et al., 1991] show separate  $H^+$

and  $O^+$  velocities and temperatures for June solstice, solar minimum conditions. For the L=2 field line they studied they predict a rather striking feature in the winter hemisphere, this feature consists of large downward proton velocities of over 2 km/s accompanied by proton temperature enhancements reaching 5600 K near dawn. We know of no previous experiments that have verified either of these simulations.

## 2 Fitting for $T_{H^+}$ and $T_{O^+}$

There are two parts to a proof that a parameter can be included in a model of a physical process and give useful measurements when the model is compared to data. The first part, discussed in this section, consists of showing that the data are sensitive to the parameter. This is accomplished by evaluating the goodness of fit for the cases when the parameter is and is not included in the model, and showing that the fit is significantly better when it is. The second part of the proof is considered later in the discussion section. It consists of eliminating other explanations for the characteristics of the data. This part can never be completed, but at the very least one must show that the data do not contain systematic errors that could mimic the proposed effect. For another discussion of a similar proof see [González and Sulzer, 1996].

We now consider what goodness of fit means, and how it can be useful in determining whether  $O^+$  and  $H^+$  temperatures can be measured independently. Consider the equation for  $\chi^2$ , the sum of the squared differences between the model and data, weighted by the statistical errors [Press *et al.*, 1994]. The model is  $y = y(x; \mathbf{a})$ , where  $x$  is the independent variable and  $\mathbf{a}$  is a vector of parameters. The error associated with data point  $j$  is  $\sigma_j$ . The  $\chi^2$  merit function is

$$\chi^2 = \sum_{j=0}^{N-1} \left[ \frac{y_j - y(x; \mathbf{a})}{\sigma_j} \right]^2.$$

For the case of a good model and known values of the  $\sigma_j$ s,  $\chi^2$  has a definite expected value that is greater than zero. That is, statistical errors prevent  $\chi^2$  from going to zero no matter how the parameters are adjusted. Furthermore, we can compute  $Q(\chi^2)$ , the probability that a particular value  $\chi^2$  or lower is obtained in a fit when the model is good. A very low value of  $Q$  (large  $\chi^2$ ) means that it is very likely that statistical errors are not the only cause of the deviation from zero, meaning the model is not complete. For good fits,  $Q$  has an expected value of .5, but it is a statistical quantity, and some variation is normal. Gaussian noise is assumed; real data can depart significantly from this assumption and *Press et al.* indicate that it is not unreasonable to accept a  $Q$  of .001 in this case when using theoretically calculated  $\sigma_j$ s. We have found that we can supplement the theoretical values with measurements, thus predicting the  $\sigma_j$ s well enough so that  $Q$  for a good model is greater than .1 on average.

For a spectrum the  $\sigma$ s are given by  $\sigma(f) = S(f)/N$ , where  $S$  is the power density at frequency  $f$ , and  $N$  is the number of independent samples included in the estimate of  $S$ . We compared these predicted values with estimates measured at the edges of the spectra. The noise in the radar signal is assumed to be Gaussian, but even a small amount of interference can slightly invalidate this assumption. Then the average of

the many independent estimates does not converge as fast as expected since the actual signal distribution is less compact than a Gaussian and so the effective value of  $N$  is less than expected. It is possible to identify problems with the  $\sigma$ s by doing a trial fit, and checking for reasonable values of  $Q$  across the entire altitude range of the experiment, especially in regions where the fits are simple. With some perseverance reasonably accurate values of the  $\sigma$ s can be obtained.

The data from the Arecibo incoherent scatter radar consist of spectral estimates at a number of ranges. We have the choice of fitting either autocorrelation functions (ACFs) or spectra; either can give the same fitted parameters. We find that it is better to monitor the fitting process with spectra because changes in the parameters correspond to local changes in the spectrum even for relatively subtle effects. We have found that this is not always true for ACFs. When the data have high signal to noise ratio it is better to fit for spectra because the statistics of the merit function for  $\chi^2$  are simpler. In applying the merit equation as stated above, it is assumed that the data points are independent, certainly very close to true for all but closely spaced points in a spectrum since spectra are essentially filter banks, ideally with no coupling. However, the lengths of the data samples are short, as determined by the length of the radar pulse, and so the filters are far from perfect and the responses must overlap in order to get all of the information about the shape of the process. We measure twice as many points as independent points, but one can scale the  $\sigma$ s by a constant in order to produce a  $\chi^2$  value that gives a valid goodness of fit in this case. However, the merit function for a fit to an ACF is quite complicated because the samples of an ACF can be highly correlated over long delays when the SNR is high. Our signal to noise ratio is high in the lower topside, and so the merit equation would need to be fully general and so very complicated [Huuskonen and Lehtinen, 1996].

Another choice that one has is to fit the data from each range independently or to reduce the number of free parameters by constraining the range variation of some parameters. This can reduce the errors in the measurement, but it can make the statistics more complicated. The fitting comparison shown below uses data from a single range.

The three panels in Figure 1 show the following: a spectrum and two fits to it from 547 km altitude (panel a), various temperature profiles (panel b), and composition profiles (panel c). In figure 1a the data are shown as unfilled black circles, while two different fitted functions are shown as gray and black lines. The horizontal scale is frequency, while the vertical scale is power in arbitrary units, using a logarithmic scale so that a wide dynamic range is visible. First consider the properties of the data. The signal to noise ratio is high out to 30 KHz near where the signal declines to the noise level. A spectrum has the property that the uncertainty at a particular point is proportional to its value at that point, where the value is the sum of both signal and noise. Over that portion of a spectrum where the signal to noise ratio is high, the errors are proportional to the signal value, while the noise determines the errors when the SNR is low. This spectrum represents a mixture of oxygen ions (responsible for the narrow feature in the spectrum from 0 to about 7 KHz) and protons (responsible for the wider part of the spectrum out to about 30 KHz), with the oxygen dominating. The SNR is above unity out to almost 30 KHz and thus is so for the hydrogen portion of the spectrum. This means that even though the hydrogen portion of the spectrum is much weaker than the oxygen part, the errors are proportionately smaller as well, and so we should be able to

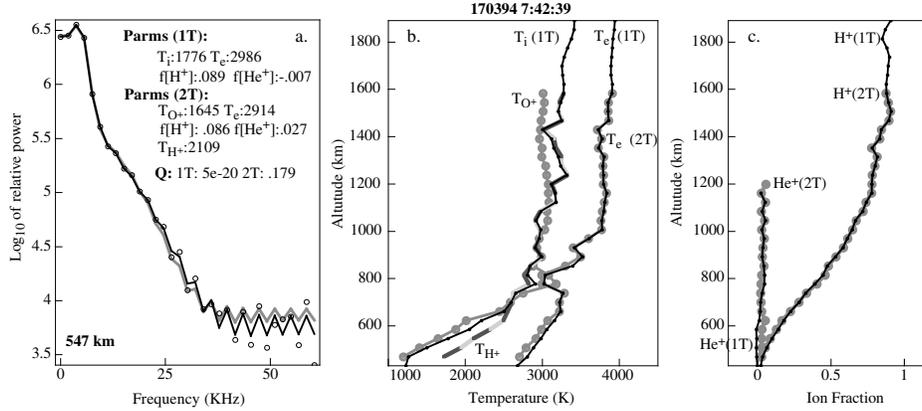


Figure 1: The three panels in this figure show a typical measured spectra and two different fits (panel a), a typical temperature profile for the two ion and three ion fits (panel b), and the associated light ion fraction profiles (panel c).

measure the characteristics of both parts almost equally well. That is, small amounts of hydrogen are sufficient as long as the SNR remains high. Of course there is some limit. If the hydrogen fraction is small enough, the error level is set by the noise, and it is not possible to measure the  $H^+$  parameters accurately.

It should also be noted that some ringing is visible between 20 and 30 KHz. This could be mistaken for noise, but it is not. This ringing is a result of the square window function applied to the ACF. Lags are not available after a certain delay; ringing could be avoided by using a tapered ACF, but this makes certain features of the spectrum less visible to the eye.

Now consider the opposite situation in which  $H^+$  is the dominant ion. Then the  $O^+$  part of the spectrum is small and sits on a large level determined by the  $H^+$ . Thus the errors in the  $O^+$  part of the spectrum are large, since they are determined by the level of  $H^+$  and we do not expect to be able to measure the temperature of the oxygen ions accurately. We will see later that this is true.

The fit in which the  $H^+$  and  $O^+$  temperatures are constrained to be the same is the gray line in Figure 1a. The fit is good in the  $O^+$  portion of the spectrum, but becomes progressively worse as the frequency is increased. The line should lie between the points, but it is too low (between 20 and 35 KHz), and then as the frequency increases more, it is too high (above 40 KHz). One of the free parameters is the baseline; minimum  $\chi^2$  is obtained by raising the baseline to compensate for the underestimate of the width. The differences might seem small, but they are very significant. The Q is less than  $10^{-19}$ , and this shows that the noise did not cause the bad fit, and we must conclude that the model is incomplete.

Although the value of Q for the single temperature shows that the weakness of the model, we must decide by other means how to modify the model in order to make it better. Thus we see the importance of plotting the data in the domain which shows

the most information to the eye. Since higher temperature implies a wider spectrum, we strongly suspect on the basis of Figure 1a that a higher  $H^+$  temperature will give a better fit, and thus we have the motivation to extend the model to allow this parameter to be independent. This would not be so obvious if we examined the ACF.

The black line in Figure 1a is the two-ion temperature fit; it meets the condition of falling amongst the data points rather than to one side of them, but it is more important that the value of  $Q$  is reasonable (.18). Based on this fit, and many others like it, we conclude that it is necessary to allow for two ion temperatures at least under some conditions, and that this is sufficient freedom in the model. That is, although  $He^+$  might have a temperature different from the other two, our data are not sufficiently sensitive for this effect to show up with such a small fraction of  $He^+$ . This is an important point, and it means two things. First, we cannot measure the Helium ion temperature in this case, and second, the effect of an independent  $He^+$  temperature in the model is truly negligible. These two are intimately linked, but the result is that even if helium ions are of a different temperature, say somewhere between  $O^+$  and  $H^+$  neglecting this in the model in no way affects our results for the other parameters.

Next we consider range profiles of the data. These profiles (Figures 1b and 1c) show results that are typical of early morning, and also similar to results in the late afternoon. Note that the temperature results have been affected by interference near 800 km. The  $H^+$  temperature (line with alternating sections of dark and light gray) is as much as several hundred degrees above that of  $O^+$  (lower temperature gray line with open circles), and the difference begins at the bottom of measurement range and declines with increasing altitude. The ion temperature measured in the single temperature fit (lower temperature black line) lies between the two measured in the two temperature fit, lying closer to that of the dominant ion. Note that the electron temperature is a bit high in the single temperature fit (higher temperature black line). At the higher altitudes it appears that the  $O^+$  temperature is higher than the  $H^+$  temperature. This is apparent only. The  $O^+$  temperatures above 1000 km or so (where the  $O^+$  ion are less numerous than  $H^+$ ) are not reliable. (Note that the  $H^+$  temperature is the same as the one ion temperature fit.) The signal to noise ratio decreases rapidly with altitude, and as the errors in certain parameters become large, constraints (e.g. [Erickson and Swartz, 1994], [González and Sulzer, 1996]) are introduced in order to prevent unrealistic extreme values in the noisy parameter from destroying the fit and thus making it impossible to measure the other parameters. This situation is easily recognized because the error in the noisy parameter becomes very nearly equal to the sigma assigned to the constraining value. This has happened above 1000 km and so we know that the  $O^+$  temperature is not a good indication of the value at these altitudes. The other parameters are not affected by the erroneous assumption of the  $O^+$  temperature; note that their values are the same as obtained with the single ion temperature fit.

As indicated earlier, using a fit with insufficient freedom can result in errors in the values of other parameters, even ones that might not seem to be related to the one that is free. Figure 1c shows the composition parameters, that is the fractions of  $H^+$  and  $He^+$  for both the single and double temperature fits. It can be seen that there are no significant errors in the hydrogen. This is not surprising, as it is extremely easy to measure the hydrogen fraction [González, 1994]; it can usually be estimated by eye

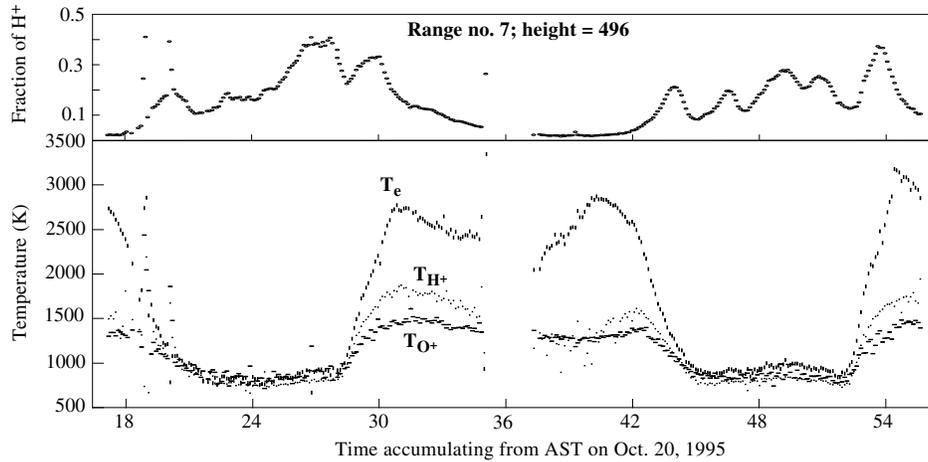


Figure 2: Time series of the  $H^+$  fraction at 496 km altitude (top panel) and the associated  $T_e$ ,  $T_{H^+}$  and  $T_{O^+}$  (bottom panel). The horizontal axis is local time, times greater than 24 refer to the next days (October 21 and 22, 1995).

with good accuracy. On the other hand, helium is more difficult, and we see that the helium is biased in the negative direction in the single temperature fit. The effect is small, being not too much larger than our measurement accuracy, but it is real as it is frequently seen and was noticed before the two temperature fits were introduced, although at the time the cause was not known.

### 3 The Diurnal Variation of the Temperatures

Figure 2a shows the fraction of  $H^+$  ions versus time at 496 km starting on October 20, 1995. We have chosen a height to display at which the hydrogen fraction varies significantly over the diurnal cycle in order to provide a fairly rigorous test for our fitting technique and illustrate its limitations. The  $H^+$  fraction falls under 3% in the afternoon at this altitude. The point we want to show is that if the  $H^+$  temperature measurement is sensitive to the  $H^+$  fraction, then how small a fraction of  $H^+$  is useful for determining a temperature? The answer to this question can be found in the next panel.

Figure 2b shows the temperatures versus time at 496 km starting on October 20, 1995. We see that the variation of the proton temperature is similar to that of  $T_e$ , rising significantly above the oxygen ion temperature at sunrise and then declining later in the morning. We are missing some data, but we can see that in the afternoon when the  $H^+$  fraction is under 3%, the measurement becomes very noisy and is not of much use. Then the proton temperature rises again late in the afternoon, falling at sunset to somewhat under the oxygen ion temperature. The daytime behavior is reasonable and we will compare it to the Sheffield model. The nighttime results are surprising, since

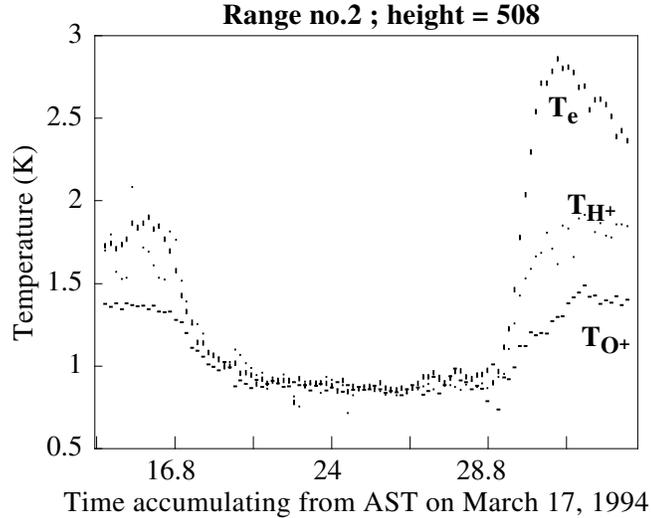


Figure 3: The same as Figure 2 bottom panel, but for March 17, 1994

we are not aware of any simulations that predict  $H^+$  to be cooler than  $O^+$  during the night. It is necessary to examine additional nights to see if this effect is real or just an artifact.

Figure 3 shows the temperatures versus time at 508 km starting on March 17, 1994. We see that on this night, all three temperatures are the same from the period well after sunset until sunrise. This behavior is expected; this does not prove that the data in the previous figure are incorrect, but it is a good indication that there may be some small bias. The data in this figure were taken using a well-established technique, while that of the previous figure were taken using a new technique. It is possible that the analysis program contains a small error. Comparing these two data sets, which have similar solar flux levels and are both taken near equinox conditions, we notice that for both the early morning behavior is essentially the same,  $T_e \sim 3000$  K, and  $T_{O^+} \sim 1500$  K with minimum daytime  $T_e/T_{H^+}$  near noon. The afternoon temperatures are different on these two days. On March 17, 1994 the  $T_{H^+}/T_e$  values are quite a bit lower than on October 20, 1995 even though  $T_{O^+}$  is similar on both days.

## 4 Discussion and Conclusions

The topside ionosphere changes slowly enough with altitude so that when one uses a  $500 \mu s$  radar pulse, one expects that the length of the pulse does not cause significant distortion of the spectra. However, there must be some level of distortion, and as more sensitive fits are introduced it is a good idea to check to make sure that there is no significant effect. This has been done. Simulated data with the two ion temperatures the same were produced, including the effect of the range smearing resulting from the radar pulse length. The data were analyzed and no significant temperature differences

were generated.

It is also necessary to consider the possible effect of large  $H^+$  drifts in generating an apparent, but false, temperature difference. Currently, we measure separate  $O^+$  and  $H^+$  drifts by analyzing delay lags in the imaginary part of the ACF, and it is apparent the temperature difference and the drifts are not so well correlated as to indicate a strong instrumental coupling. We have examined this problem and determined that a 300 m/s velocity (this would be the highest value we would expect for the altitudes and times where we measure different ion temperatures) would cause a false temperature of nearly 40 k. This is small, but not negligible, and in the future we may be able to correct for this. It may be best to incorporate the drifts into the NLLS fitting together with the other parameters.

It is possible (and necessary during the day) to measure separate  $H^+$  and  $O^+$  temperatures in the lower topside ionosphere over Arecibo. For the equinox solar minimum conditions examined, the  $H^+$  temperatures are higher than the  $O^+$  temperatures during the daytime and have a variation similar to that of  $T_e$ .  $T_{H^+}$  reaches a peak in the morning, decreases around noon and has another peak in the afternoon.

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The Arecibo Observatory is part of the National Astronomy and Ionosphere Center, which is operated by Cornell University under a cooperative agreement with the National Science Foundation.