This writeup is intended to clarify the relationship between polarimetry done with a correlator and continuum polarimetry done with a single-channel system. First I derive the correlation functions for a monochromatic signal and then a continuum signal. These show explicitly how the Q and U Stokes parameters are related to the symmetric and antisymmetric components of the cross-correlation function of the LHCP and RHCP voltages.

**Monochromatic, Linearly Polarized Signals**

Consider a monochromatic, linearly polarized signal at RF:

\[ \hat{E}(t) = c \cos(\omega t)\hat{x} + s \cos(\omega t)\hat{y}. \]  

(1)

where \( c = \cos \chi \) and \( s = \sin \chi \) and \( \chi \) is the position angle. The corresponding components of circular polarization are

\[ E_{L,R}(t) = E_x(t) \pm E_y(t - t_{1/4}) \]  

(2)

where \( t_{1/4} \equiv \pi/2\omega_0 \) is a delay equal to one quarter cycle at the center frequency \( \omega_0 \). The IF signals are (with implicit filtering off of the upper sideband)

\[ L_{IF}, R_{IF} = E_{L,R}(t) \cos(\omega_{LO} t) \]  

(3)

and the baseband voltages are (again with implicit filtering)

\[ \ell(t), r(t) = L_{IF}, R_{IF}(t) \cos(\omega_{LO_2} t). \]  

(4)

The autocorrelation functions (acfs) of the baseband fields are

\[ \langle \ell(t)\ell(t + \tau) \rangle = \frac{1}{2} \cos(\delta \omega \tau) [c^2 + s^2 + 2cs \cos(\omega t_{1/4})] = \frac{1}{2} \cos(\delta \omega \tau) \]  

(5)

\[ \langle r(t)r(t + \tau) \rangle = \frac{1}{2} \cos(\delta \omega \tau) [c^2 + s^2 - 2cs \cos(\omega t_{1/4})] = \frac{1}{2} \cos(\delta \omega \tau) \]  

(6)

from which two of the Stokes parameter correlations may be defined:

\[ I(\tau) = \langle \ell(t)\ell(t + \tau) \rangle + \langle r(t)r(t + \tau) \rangle = \cos(\delta \omega \tau) \]  

\[ V(\tau) = \langle \ell(t)\ell(t + \tau) \rangle - \langle r(t)r(t + \tau) \rangle = 0. \]  

(7)

The cross correlation function (ccf) between \( \ell \) and \( r \) yields \( L(\tau) \equiv Q(\tau) + U(\tau) \):

\[ L(\tau) = 2\langle \ell(t)r(t + \tau) \rangle = [(c^2 - s^2) \cos(\delta \omega \tau) - 2cs \sin(\delta \omega \tau) \sin(\omega t_{1/4})], \]  

\[ = \cos 2\chi \cos \delta \omega \tau - \sin 2\chi \sin \delta \omega \tau \]  

\[ = \cos(\delta \omega \tau + 2\chi) \]  

(8)
where $\delta \omega \equiv \omega - \omega_{LO_1} - \omega_{LO_2}$ is the baseband frequency of the monochromatic signal. Lines 1-2 of Eq. (8) explicitly show symmetric and antisymmetric parts of $L(\tau)$. When transformed to the frequency domain, they correspond to $Q(\tilde{\omega})$ and $U(\tilde{\omega})$, respectively. The position angle is found in the usual way to be

$$\chi_{\tilde{\omega}} = -\left(\frac{1}{2}\right) \tan^{-1} \frac{U(\tilde{\omega})}{Q(\tilde{\omega})}.$$  \hspace{1cm} (9)

**Linearly Polarized Continuum Signal**

Now consider a continuum signal, again with 100% polarization, analyzed in a total bandwidth $B$. We assume there is no Faraday rotation across the bandwidth (see below). Stokes parameters may be found by integrating the monochromatic result over frequency because the SP’s are variance-like quantities and the frequency components are statistically independent (variances add). Performing integrals like

$$I(\tau) = \int_0^B d\delta \omega I_m(\tau),$$

$$L(\tau) = \int_0^B d\delta \omega L_m(\tau),$$  \hspace{1cm} (10)

where $I_m, L_m$ are the monochromatic results from Eq. (7)-(8) [that depend on the frequency $\delta \omega$], we find that

$$I(\tau) = \frac{\sin B\tau}{\tau}$$

$$L(\tau) = \cos 2\chi \left(\frac{\sin B\tau}{\tau}\right) + \sin 2\chi \left(\frac{\cos B\tau - 1}{\tau}\right)$$

$$V(\tau) = 0.$$  \hspace{1cm} (11)

When transformed to the frequency domain, Eq. (11) yield the Stokes parameters vs. frequency. In this case, the SPs are independent of frequency and it may be seen that a linearly polarized continuum source with arbitrary position angle is representable with correlation functions as we have defined them in Eq. (5) - (8).

**Elliptically Polarized Signals**

A monochromatic signal with arbitrary elliptical polarization is handled as above. General expressions that include differential timing delays, Faraday rotation, and LO phase offsets may be found in Eq. (C3)-(C4) in my 1988 memo, *Polarimetry with the 40 MHz Correlator*. Similarly, noise-like signals with arbitrary polarization may be analyzed by integration of the monochromatic results, as we did above.
Single Channel Polarimetry

By single channel polarimetry, I mean a system where only a single lag of the auto- and cross correlations is computed. This might be effected with analog multipliers rather than a digital correlator. In this case, one must explicitly calculate the ccf between the RHCP and LHCP components with a 90° phase shift as well as without a phase shift. This may be seen by using Eq. (1)-(2) and calculating the cross correlations (for a monochromatic signal)

\[
2\langle E_R(t)E_L(t + \tau) \rangle = \cos 2\chi \cos \omega \tau + \sin 2\chi \sin \omega \tau
\]

\[
2\langle E_R(t)E_L(t + \tau - t_{1/4}) \rangle = \cos 2\chi \sin \omega \tau - \sin 2\chi \cos \omega \tau.
\]

At zero lag (\(\tau = 0\)), the unshifted correlation (Eq. 12) gives \(\cos 2\chi\) while the shifted correlation (Eq. 13) gives \(\sin 2\chi\); both \(\cos 2\chi\) and \(\sin 2\chi\) are needed to solve for the position angle without ambiguity.