Notes Relevant to Spectral-Line Calibration

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1 Relevant Definitions

Antenna Temperature (T_A): Suppose that a telescope is observing a point source of flux density, S. Now suppose that the telescope feed is replaced by a resistor whose temperature, T, is adjusted until the noise power received from the resistor, ($P = k\mathrm{T}\Delta\nu$, where k is Boltzmann's constant = 1.38×10^{-23} Joule K^{-1}, and $\Delta\nu$ is the receiver bandwidth), equals that previously received from the point source. If the temperature of the resistor is then T_A, this is known as the antenna temperature due to the source.

System Temperature (T_{sys}) is the total system noise expressed as an Antenna Temperature. At Arecibo, T_{sys} is essentially a monotonic function of zenith angle, za. To first order, T_{sys} is;

$$\mathrm{T}_{sys}(za) = \mathrm{T}_{rx} + \mathrm{T}_{gr}(za) + \mathrm{T}_{atm}(za) + \mathrm{T}_{cel}(\alpha, \delta, za) \quad (1)$$

Where, a) T_{rx} is the receiver temperature; b) T_{gr} is the contribution from the ground due to spillover, vignetting, etc. This is likely to also be a weak function of azimuth; c) T_{atm} is the contribution from the atmosphere which will generally increase as frequency increases. At wavelengths shorter than about λ11 cm, especially in adverse weather conditions, T_{atm} can vary significantly on the time scale of a few tenths of a second; d) T_{cel} is the celestial contribution, which can be roughly expressed as;

$$\mathrm{T}_{cel} = \mathrm{T}_{CMB} + \mathrm{T}_{BG}(\alpha, \delta) + \mathrm{T}_{source}(za) \quad (2)$$

Where T_{CMB} is the contribution from the Cosmic Microwave Background (~ 2.7 K), T_{BG} is the contribution from the celestial "background" radiation such as the galactic "background" emission and individual confusing sources, and T_{source} is the contribution of the source itself which will be a function of zenith angle.

Telescope Gain (G) is the telescope response to a point source. At Arecibo, this is usually expressed as the antenna temperature, T_A, due to a point source of flux density 1 Jy lying at the peak of the telescope beam, and has units of K/Jy. G has a marked, monotonic variation with zenith angle at Arecibo, i.e. $G = G(za)$. At present it also has an azimuth dependence which becomes more marked as frequency is increased. At least part of this azimuth dependency is thought to be due to imperfections in the figure of the primary reflector that vary around the surface. These surface imperfections are expected to be rectified in the coming months.

[**N.B.** It should be noted that the above definition of gain is intimately connected with the Directivity of the telescope, D, the ratio of the radiated intensity at the beam center to the average radiated power (with the telescope transmitting.) D is related to the effective telescope area, A_{eff}, by $D = 4\pi A_{eff}/\lambda^2$. In equation (5) below it is shown that $A_{eff} = 2k\mathrm{G}$, so $D = 8\pi k\mathrm{G}/\lambda^2$, and thus $D \propto G$.]

System Equivalent Flux Density (SEFD) is the system temperature expressed as an equivalent point-source flux density. It has the units of Jy. It is not an independent quantity, but depends on the system temperature and telescope gain via;

$$\mathrm{SEFD}(za) = T_{sys}(za) \;/\; G(za) \qquad (3)$$

It is to be noted that SEFD can be estimated rather accurately by measurements of a point source of known flux density without recourse to secondary standards such as noise-diode calibrations. The major source of error in the measured SEFD will usually be the uncertainty of the flux density of the point source.

To illustrate the above, if a point source of known flux density, $S(\nu)$, is measured using ON/OFFs, cross-scans, or turret-scans, then if ON is the total signal when pointing directly at the source, and OFF is the signal at an adjacent off-source position (assumed to be "blank sky"), then;

$$\mathrm{SEFD}(za) = S(\nu) * \mathrm{OFF} \;/\; [\mathrm{ON} - \mathrm{OFF}] \qquad (3a)$$

Note that to obtain the total signals, ON and OFF, the "true zero" of the total-power signal needs to be known (e.g. via the correlator or appropriate setting of the continuum system).

Note too that no noise-diode measurement needs be taken, though its presence makes simultaneous computation of T_{sys} and G possible. If indeed no noise-diode measures are made, T_{sys} values relative to that at some preferred zenith angle za_0 can still be calculated by $T_{sys}(za)/T_{sys}(za_0) = \mathrm{OFF}(za)/\mathrm{OFF}(za_0)$. Then, once the SEFD has also been computed, the normalized telescope gain, $G(za)/T_{sys}(za_0)$, can be calculated from,

$$\frac{G(za)}{T_{sys}(za_0)} = \frac{T_{sys}(za)/T_{sys}(za_0)}{\mathrm{SEFD}(za)} \qquad (4)$$

If $T_{sys}(za_0)$ is known, then $T_{sys}(za)$ and $G(za)$ can be expressed in units of K and K/Jy respectively.

As there may be azimuth dependencies to both T_{sys} and G at present, there also also be an azimuth dependency to SEFD.

Effective Area (A_{eff}) is the equivalent area over which radiation would have to be collected with perfect efficiency to give the same telescope gain as the actual telescope. For radiation from a point source of flux density, S, producing an antenna temperature, T_A;

$$\frac{1}{2} S A_{eff} = k T_A$$

So, $$A_{eff} = 2k\ G \tag{5}$$

Where, $$G = T_A/S$$

As an example, G = 8 K/Jy would result from an effective area of 22,080 m^2, which would be a circular aperture of diameter 168 m. Note that A_{eff} is also a function of zenith angle (and increasingly azimuth at higher frequencies) at Arecibo.

2 Possible Calibration Approaches

Using a Standard SEFD Curve: If an ON/OFF measurement has been made, then;

$$\text{Signal(Jy/beam)} = \text{SEFD}(za) * \frac{[\text{ON}-\text{OFF}]}{\text{OFF}} \tag{6}$$

SEFD(za) could be taken from a standard curve, or be measured especially for the individual measurement using a continuum source of known flux density and applying equation(3a). In the above, we assume that T_{sys}/G is independent of frequency across the band, which is probably not true due to the presence of standing waves, etc.

N.B. T_{sys} in the OFF position at a given zenith angle may not be the same as when the standard curve was measured due to a) the presence of a confusing source "in the OFF", b) the increasingly large continuum "background" contribution at lower galactic latitudes and frequencies, and/or c) secular changes in T_{sys} since the SEFD curve was last measured. However, these effects can often be corrected for by estimating the increment to the SEFD from $\Delta\text{SEFD} = \Delta T_{sys}/G(za)$.

Using the Noise Diode Calibration and a Standard Gain Curve: Suppose CAL_{ON} and CAL_{OFF} measurements are made along with an ON/OFF source measurement. Also suppose that the antenna temperature of the noise diode signal (i.e. $CAL_{ON} - CAL_{OFF}$) is T_{cal}, a value which is the same at all zenith angles. Then, the system temperature is given by;

$$T_{sys}(K) = T_{cal} * \frac{CAL_{OFF}}{[CAL_{ON} - CAL_{OFF}]}$$

and the antenna temperature of the source is;

$$\mathrm{T}_{source}(\mathrm{K}) = \mathrm{T}_{cal} * \frac{\mathrm{CAL}_{OFF}}{[\mathrm{CAL}_{ON} - \mathrm{CAL}_{OFF}]} * \frac{[\mathrm{ON} - \mathrm{OFF}]}{\mathrm{OFF}}$$

(where CAL_{OFF} can be taken to be essentially equal to OFF.)

Thus;

$$\mathrm{Signal(Jy/beam)} = \frac{\mathrm{T}_{cal}}{\mathrm{G}(za)} * \frac{\mathrm{CAL}_{OFF}}{[\mathrm{CAL}_{ON} - \mathrm{CAL}_{OFF}]} * \frac{[\mathrm{ON} - \mathrm{OFF}]}{\mathrm{OFF}} \qquad (7)$$

So if G(za) and T_{cal} are known, the observation can be simply converted into Jy/beam using equation (7).

[**N.B.** It should be noted that;

$$\mathrm{SEFD}(za) = \frac{\mathrm{T}_{cal}}{\mathrm{G}(za)} * \frac{\mathrm{CAL}_{OFF}}{[\mathrm{CAL}_{ON} - \mathrm{CAL}_{OFF}]}$$

Therefore, equation (7) reduces to;

$$\mathrm{Signal(Jy/beam)} = \mathrm{SEFD}(za) * \frac{[\mathrm{ON} - \mathrm{OFF}]}{\mathrm{OFF}}$$

This is identical to equation (6). Thus, this method is not in any way in conflict with that using a standard SEFD curve, and the user can select whichever of the two they feel represent the present status of the system best. This second method does not assume that the T_{sys} is the same as when a standard SEFD curve was measured. Nevertheless, it does assume that $\mathrm{G}(za)$ has remained unchanged since the gain curve used was last measured, and that T_{cal} does not vary with time.]

Using a Continuum Calibrator of Known Flux Density: When applied to spectral-line calibration, both of the above approaches ignore likely changes across the passband, i.e. the same functions for SEFD(za) and G(za), plus the value for T_{cal}, are assumed to apply across the whole passband. In practise this is not the case. For example, if the line emitter has significant continuum emission associated with it, or a significant continuum source is present in either the ON or OFF beam, or the line is wide, then standing waves modulating the effective passband response may result in poor baselines. This can be largely overcome, (though at a theoretical cost in sensitivity/observing time – see below), by interspersing identical ON/OFF measurements of a continuum "calibrator" source, covering as near as possible the same tracks across the primary reflector (i.e. the same azimuth-zenith angle tracks). The calibrator source would ideally have a well-established flux density (i.e. not be significantly variable), and preferably be of similar flux density to that of the target field. The tolerance to this latter point presently needs evaluation.

For identical ON/OFF pairs on a target and a calibrator source observed quasi-simultaneously over the same range of azimuth and zenith angle,

$$\mathrm{Signal(Jy/beam)} = \mathrm{S}_{c\_cal} * \frac{\mathrm{OFF}_{c\_cal}}{[\mathrm{ON}_{c\_cal} - \mathrm{OFF}_{c\_cal}]} * \frac{[\mathrm{ON} - \mathrm{OFF}]}{\mathrm{OFF}} \qquad (8)$$

Where S_{c_cal} is the flux density of the continuum calibrator source in Jy at the observing frequency.

The cost paid is best illustrated by considering the rms, $\sigma_T(K)$, for observations with a receiver of system temperature, T_{sys}, bandwidth, β, and total integration time per observing cycle, τ. For the following specified cases, the theoretical sensitivities are;

- **Total-Power Observations:** Here all the observing time is spent looking at the target. This gives, $\sigma_T = T_{sys}/\sqrt{(\beta\tau)}$.
- **"In-Band" Frequency switching:** Here the line is always in the observing band, but at each of the two frequency positions where it falls, only one half of the time is spent looking at the line. This gives, $\sigma_T = \sqrt{2}T_{sys}/\sqrt{\beta\tau}$.
- **Position Switching or "Out of Band" Frequency Switching:** Here the line is only observed for one half of the time, with noise observed all of the time. This gives, $\sigma_T = 2T_{sys}/\sqrt{\beta\tau}$.
- **Position Switching on a Target Source, and a Band-Pass Continuum Calibrator:** Here the line is observed for one quarter of the time, but noise is observed all the time. This gives, $\sigma_T = 4T_{sys}/\sqrt{\beta\tau}$. Note that T here is not just the "blank-sky" system temperature, but should include the contribution due to the continuum emission of the target and calibrator, i.e. if both the sources have a flux density, S Jy, then T_{sys} should be increased by $\Delta T_{sys}(K) = S * G(za)$.

3 Radial Velocities

Optical Velocities: Optical astronomers traditionally use wavelengths to define redshifts and radial velocities. **It is important to note that these days almost all radio astronomers do the same!** If z_{opt} is the the optically defined redshift, and v_{opt} the optically defined (non-relativistic) radial velocity for a line emitted at wavelength, λ_0, and observed at a wavelength, λ, then;

$$z_{opt} = \frac{(\lambda - \lambda_0)}{\lambda_0} = v_{opt}/c \qquad (9)$$

N.B. v_{opt} only represents a true velocity if $v \ll c$). However, it is usual to present radial velocities as $v = cz$, meaning that this "velocity" is really a "scaled redshift".

Radio Velocities: In the early days of radio astronomy, its practitioners used frequencies to define redshifts and radial velocities. If z_{rad} is the the radio defined redshift, and v_{rad} the radio defined (non-relativistic) radial velocity for a line emitted at frequency, ν_0, and observed at a frequency, ν, then;

$$z_{rad} = \frac{(\nu_0 - \nu)}{\nu_0} = v_{rad}/c \qquad (10)$$

¿From equations (10) and (11), it follows that for a given target, $(z_{opt}/z_{rad}) = \lambda/\lambda_0$.

4 Relating T_A to T_b

In discussing this issue, the formalism of Seeger et al. (B.A.I.N., 1965, **18**, 11) is followed.

Yet More Definitions: The antenna power pattern, $f(\theta,\phi)$ is the power response to an unpolarized wave arriving from the direction (θ,ϕ), where the maximum response is normalized to be, $f(0,0) = 1$.

The antenna solid angle is the integral of $f(\theta,\phi)$ over the whole sphere, $\Omega = \int\int_{4\pi} f(\theta,\phi)d\Omega$. Note that $\Omega = 4\pi/\mathrm{D} = \lambda^2/\mathrm{A}_{eff} = \lambda^2/2k\mathrm{G}$, where G is as defined in Section 1.

The "full beam" is defined to be that part of the antenna beam which which one is interested for a particular observation, giving a full-beam solid angle of $\Omega_{\mathrm{fb}} = \int\int_{\mathrm{fb}} f(\theta,\phi)d\Omega$. (Note that if the full beam were to be Gaussian down to zero, then $\Omega_{\mathrm{fb}} = 1.133\theta_V\theta_H$, where θ_V and θ_H are the orthogonal half-power widths.) Now the full-beam efficiency is defined as,

$$\eta_B = \Omega_{\mathrm{fb}}/\Omega \quad < 1 \qquad (11)$$

The brightness temperature of the sky, T_b in a given direction is the temperature of a black body situated in that direction for which the brightness of the thermal radiation would equal the brightness of the sky, B. For the Rayleigh-Jeans approximation, $\mathrm{T}_b = \lambda^2\mathrm{B}/2k$.

We can define a full-beam brightness temperature, T_{fb}, to be the weighted mean of the brightness temperature over the full beam, i.e.,

$$\mathrm{T}_{\mathrm{fb}} = \frac{1}{\Omega_{\mathrm{fb}}}\int\int_{\mathrm{fb}} f(\theta,\phi)\mathrm{T}_b(\theta,\phi)d\Omega \qquad (12)$$

Now the observed Antenna Temperature due to the whole sky, T_A, is,

$$\mathrm{T}_A = \frac{1}{\Omega}\int\int_{4\pi} f(\theta,\phi)\mathrm{T}_b(\theta,\phi)d\Omega \qquad (13)$$

So,

$$\mathrm{T}_A = \frac{\eta_B}{\Omega_{\mathrm{fb}}}[\int\int_{fb} f(\theta,\phi)\mathrm{T}_b(\theta,\phi)d\Omega + \int\int_{4\pi - fb} f(\theta,\phi)\mathrm{T}_b(\theta,\phi)d\Omega]$$

If the full beam is a very small fraction of 4π steradians, we can take the second term as being constant over a measurement. This then gives,

$$\mathrm{T}_A = \eta_B\mathrm{T}_{\mathrm{fb}} + \text{ constant} \qquad (14)$$

Thus the constant of proportionality between the antenna and full-beam temperatures is η_B.

As $\Omega = \lambda^2/2k\mathrm{G}$, and Ω_{beam} can be obtained from integrating a "beam map" made over the area of the adopted full beam, η_B can be derived via equation (11), and hence measured antenna temperatures turned into full-beam brightness temperatures.

N.B.: As $\mathrm{G} = \mathrm{G}(za)$, and $\Omega_{\mathrm{fb}} = \Omega_{\mathrm{fb}}(za)$, η_B will also be a function of zenith angle unless $\Omega_{\mathrm{fb}}(za) \propto 1/\mathrm{G}(za)$.

One "full beam" whose solid angle is well defined is the "Moon beam", which is that area of the beam lying within a circle of the same radius as the Moon. If the true brightness temperature of the Moon at the observing frequency (a well tabulated function) is $\mathrm{T}_b(\mathrm{Moon})$, and an antenna temperature of $\mathrm{T}_A(\mathrm{Moon})$ is measured for the Moon, then $\eta(\mathrm{Moon}) = \mathrm{T}_A(\mathrm{Moon})/\mathrm{T}_b(\mathrm{Moon})$. In addition, now that Ω and η_{Moon} are known, Ω_{Moon} can be derived from equation (11).