Absorptive Loss and Antenna Performance and Measurements

Paul F. Goldsmith

February 26, 2002

The purpose of this memorandum is to clear up some confusion about the effect of antenna loss (ohmic absorption) on antenna performance, and to describe how this effect enters into commonly used parameters describing antenna characteristics.

The first section discusses how ohmic loss affects standard antenna terms. The second section addresses specific calibration schemes and how they relate to the way in which ohmic loss enters into different antenna parameters.

1 Antenna Parameters

We wish to maintain the fundamental relationship between the antenna effective area, A_e, and the power collected from an unpolarized plane wave of flux density S by a single mode and single polarization feed/transmission line system

$$P = \frac{1}{2} A_e \, S \, \delta\nu \tag{1}$$

where $\delta\nu$ is the bandwidth.

If the antenna, the feed system, or both have ohmic loss, this should logically be considered as one of the terms entering into the effective area. We could imagine either a perfectly reflective surface being coated with a lossy material, or the feed system having a non-zero attenuation. The exact details make the practical impact slightly different (as discussed in Section 2) but we can write in general that

$$A_e(\text{lossy}) = A_e(\text{lossless}) \, \epsilon_r , \tag{2}$$

where ϵ_r is the ohmic loss efficiency; $0 \leq \epsilon_r \leq 1.0$.

Another important antenna characteristic is the normalized power pattern, $P_n(\theta, \phi)$, which gives the relative response to signals from different directions. By definition, P_n has its maximum value on the boresight direction: $P_n(0, 0) = 1.0$. Due to this normalization, the normalized power pattern will not, to first order, be affected by ohmic loss. It will certainly be quite unaffected by loss in the single mode transmission line as this cannot affect the form of the aperture plane illumination or the power pattern. Ohmic loss in the feed and antenna reflectors is a more subtle issue, as it is possible to conceive of situations in which certain regions have more loss than others. Then, the form of the aperture plane illumination could be different, with the result that P_n is different as well. However, in most situations that appear reasonable, the loss will result in a uniform reduction of the magnitude of the aperture plane electric field, by a fac! tor equal to the square root of the fractional power reduction. The power reduction could be due to ohmic absorption in reflector point, to other surface coating (e.g. algae), or to partial transmission (leakage) through the surface. In all of these situations, we would expect P_n to be unaffected. As a result, the antenna solid angle, given by

$$\Omega_a = \iint\limits_{4\pi} P_n(\Omega') \, d\Omega' , \tag{3}$$

is also unaffected by the ohmic loss.

The directivity of the antenna, defined by

$$D = 4\pi/\Omega_a, \tag{4}$$

remains unchanged as well.

A very important relationship is the "antenna theorem", relating the solid angle and the effective area of any antenna. For any lossless antenna operating at wavelength λ, the antenna theorem is

$$A_e\,\Omega_a = \lambda^2. \tag{5}$$

We have seen that if we consider ohmic loss, we reduce the effective area, but as long as the form of the aperture plane illumination is unchanged, P_n and Ω_a will be unaffected. Hence, the antenna theorem must be modified, and becomes

$$A_e\,\Omega_a = \epsilon_r \lambda^2. \tag{6}$$

This is consistent with the derivation of antenna theorem presented in P. Goldsmith's lecture to the first NAIC-NRAO Single-Dish Radio Astronomy Summer School. This derivation, suggested by Dr. J. Hagen, has two steps. The first demonstrates that any antenna at a given wavelength has the same value of $A_e\,\Omega_a$. This step is unaffected by possible ohmic loss. The second step determines this constant for a simple antenna for which P_n can be explicitly calculated. In this step, if we imagine adding an absorber with fractional power transmission ϵ_r, the product $A_e \Omega_a$ will be modified to be equal to $\epsilon_r \lambda^2$, rather than λ^2. Thus, the antenna theorem for antennas with ohmic loss is as given by previous equation.

The antenna gain is defined as the power received from a plane wave, compared to the power received by a lossless isotropic antenna. For the latter, with $\Omega_a = 4\pi$, we have $A_e(\text{iso}) = \lambda^2/4\pi$. The general antenna gain is then

$$G = P/P(\text{iso}) = A_e/A_e(\text{iso}) = 4\pi A_e/\lambda^2 \tag{7}$$

Evidently, since A_e is proportional to ϵ_r, so is the antenna gain. This is the key difference between directivity and gain.

The antenna temperature is a surrogate for the power received in bandwidth $\delta\nu$, related through

$$P = kT_A\,\delta\nu \tag{8}$$

For an extended source with temperature distribution $T(\Omega')$, we have in the Rayleigh-Jeans limit

$$P = \frac{1}{2} A_e \delta\nu \iint P_n(\Omega') \frac{2kT(\Omega')}{\lambda^2}\, d\Omega', \tag{9}$$

and thus

$$T_A = \frac{A_e}{\lambda^2} \iint P_n(\Omega') T(\Omega') d\Omega'. \tag{10}$$

Note that since A_e includes ohmic loss, all antenna temperatures are reduced accordingly. If one wants to get rid of A_e from equation 11, it is important to use equation 6 rather than 5 if ohmic loss is significant, which gives us

$$T_A = \frac{\epsilon_r}{\Omega_a} \iint P_n(\Omega')\, T(\Omega')\, d\Omega'. \tag{11}$$

2 Calibration Procedures and Antenna Parameters

2.1 Cm-Wavelength Calibration

One approach widely used at cm–wavelengths, as described in K. O'Neil's Summer School lecture, is to use a noise diode as a secondary standard. We assume it is calibrated against a primary (generally thermal) standard. Using the noise diode calibrates the system relative to the point where the noise diode signal is coupled into the receiver transmission line.

Comparing the net signal "on source – off source" to the net signal "diode on – diode off" gives the source antenna temperature defined relative to the noise diode signal injection point.

Evidently, any attenuation of the signal before this point affects the antenna temperature measured, and this includes the signal from an astronomical calibration source used to measure the antenna performance.

2.2 System Temperature

A variation on this approach enables one to determine the system temperature, which we define here as

$$T_{\rm sys} = T_{\rm rx} + T_{\rm emis} + T_{\rm atm} + T_{\rm bg} \tag{12}$$

where $T_{\rm rx}$ is the receiver noise temperature,
$T_{\rm emis}$ is the noise temperature of any emission,
$T_{\rm atm}$ is the noise temperature contributed by the atmosphere,
$T_{\rm bg}$ is any other background (Galactic or Cosmic).

We let output of the system be denoted V, and the system gain, bandwidth, and other factors are subsumed into the constant α so

$$V = \alpha T\,. \tag{13}$$

Thus, looking at sky (off source) with noise diode on and off gives us, respectively

$$V_{\rm nd-off} = \alpha\, T_{\rm sys} \tag{14}$$

$$V_{\rm nd-on} = \alpha\, (T_{\rm sys} + T_{\rm nd}) \tag{15}$$

so that

$$\alpha = \frac{V_{\rm nd-on} - V_{\rm nd-off}}{T_{\rm nd}}\,. \tag{16}$$

The system temperature is just

$$T_{\rm sys} = \left[\frac{V_{\rm nd-off}}{V_{\rm nd-on} - V_{\rm nd-off}}\right] T_{\rm nd}\,. \tag{17}$$

This is useful to know as it determines e.g. fluctuations in the output of the radiometer.

Now if we observe a source

$$V_{\rm on-source} = \alpha\,(T_{\rm sys} + T{\rm source}) \tag{18}$$
$$V_{\rm off-source} = \alpha\,T_{\rm sys} \tag{19}$$

so that

$$T_{\rm source} = \left[\frac{V_{\rm on-source} - V_{\rm off-source}}{V_{\rm nd-on} - V_{\rm nd-off}}\right] T_{\rm nd}. \tag{20}$$

The measured antenna temperature from the source still is referred to the noise diode injection point. The signal from a source will still be reduced by any ohmic loss before that point. The ohmic loss produces emission, which adds to the system temperature, as seen in equation 12, and is reflected in the measured value of this quantity. The attenuation of the signal by the ohmic loss is **not** accounted for.

2.3 Chopper Wheel Calibration

At millimeter wavelengths, a different system has evolved, generally called the "chopper wheel" or "Penzias-Burris" calibration method. It involves placing an ambient temperature absorber over the feed to give "ambient" input, and using the sky (off source) as the "sky" input.

If you assume that the absorption and emission originate at the same temperature as the ambient temperature absorber, we get the outputs

$$V_{\rm amb} = \alpha\,(T_{\rm rx} + T_{\rm amb}) \tag{21}$$
$$V_{\rm sky} = \alpha\left(T_{\rm rx} + T_{\rm amb}\,(1 - e^{-\tau})\right). \tag{22}$$

Here τ is the total optical depth of the atmosphere PLUS the optical depth of the telescope. This somewhat unusual definition reflects the fact that you can consider the telescope emission as being associated with a particular optical depth and fractional transmission

$$\epsilon_r = e^{-\tau_{\rm tel}}\,. \tag{23}$$

Thus, we can write

$$\tau = \tau_{\rm tel} + \tau_{\rm atm}\sec z\,. \tag{24}$$

If we form the difference between $V_{\rm amb}$ and $V_{\rm sky}$, we see that

$$V_{\rm amb} - V_{\rm sky} = \alpha\,T_{\rm amb}\,e^{-\tau} \tag{25}$$

and

$$\alpha = \frac{V_{\rm amb} - V_{\rm sky}}{T_{\rm amb}}\, e^{\tau} \tag{26}$$

An on-source minus off-source pair gives us

$$V_{\rm on-source} - V_{\rm off-source} = \alpha\, T_{\rm source}\, e^{-\tau} \tag{27}$$

so that

$$T_{\rm source} = \frac{V_{\rm on-source} - V_{\rm off-source}}{\alpha e^{-\tau}} \tag{28}$$

$$T_{\rm source} = \left[\frac{V_{\rm on-source} - V_{\rm off-source}}{V_{\rm amb} - V_{\rm sky}}\right] T_{\rm amb}\ . \tag{29}$$

Thus, with this calibration scheme, both the atmospheric AND telescope optical depths ARE CORRECTED FOR. Then, if we consider observing a standard source for antenna calibration, the ohmic loss has been included as part of the atmosphere, so that if it is non-zero, we are getting an overly high value of A_e. The only observational way to disentangle $\tau_{\rm tel}$ and $\tau_{\rm atm}$ is to do a "sky dip", i.e. varying sec z, and extrapolating measured $T_{\rm sys}$ or τ to sec $z = 0$, giving the value of $\tau_{\rm tel}$. This is not possible at Arecibo due to limited zenith angle coverage. If you do this, you can determine the ohmic loss, and include it in the antenna effective area.

3 Summary

The noise diode calibration technique does not remove the contribution of ohmic loss to the effective area. Measurement of a standard source should give A_e including ohmic loss. Determining the system temperature using noise diode calibration includes emission from ohmic loss but still does not specifically correct for its reduction of the effective area, unlike the chopper wheel calibration technique.