INSTRUMENTAL POLARIZATION: POLARIZATION ISOLATION & ERRORS IN STOKES PARAMETERS

Jim Cordes & Joe Lazio

22 May 1996

Summary

The purpose of this memo is to:

(1) demonstrate how polarization isolation is related to errors in Stokes parameters;

(2) estimate a tolerable level of errors in Stokes parameters due to amplitude and phase mismatch in and before the hybrid;

(3) emphasize that Stokes parameters will have to be corrected after the fact no matter how good the hardware.

(4) estimate that isolation of $\sim$ 25 dB is needed, corresponding to phase errors $\sim 5^\circ$ and power ratio $\sim$ 0.8 dB; it probably is *not* worth making heroic efforts to obtain increased isolation, because software correction can better address the totality of systematic errors that contaminate raw polarization data.

Stokes Parameter Matrix

In our 1992 Memo (*Instrumental Polarization: Cross Coupling and Hybrid Conversion Issues*) we showed that the measured (primes) and true Stokes parameters are related in the following way:

$$\begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} = \begin{pmatrix} \overline{G} & \frac{\Delta G}{2} & 0 & 0 \\ \frac{\Delta G}{2} & \overline{G} & 0 & 0 \\ 0 & 0 & \gamma \cos\phi_\gamma & -\gamma \sin\phi_\gamma \\ 0 & 0 & \gamma \sin\phi_\gamma & +\gamma\cos\phi_\gamma \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}, \qquad (1)$$

where

$$\overline{G} = \frac{1}{2}(G_x + G_y)$$

$$G_{x,y} = |g_{x,y}|^2$$

$$\Delta G = G_x - G_y$$

$$\gamma = (G_x G_y)^{1/2}$$
$$\phi_\gamma = \phi_x - \phi_y - \delta\phi,$$

$g_{x,y}$ are the complex gains of the x, y signals going into the hybrid, $\phi_{x,y}$ is the phase of the complex gain $g_{x,y}$ and $\delta\phi$ is the phase error in the hybrid.

Polarization Error

Consider pure RHCP of unit amplitude incident on the telescope. The true and measured Stokes parameters are (using the $V = LHCP - RHCP$ convention)

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \qquad \begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} = \begin{pmatrix} \overline{G} \\ \Delta G/2 \\ -\gamma \sin\phi_\gamma \\ -\gamma\cos\phi_\gamma \end{pmatrix}. \tag{2}$$

The polarization error, defined as the error in the polarized intensity ($\Delta I_{pol} = [\Delta Q^2 + \Delta U^2 + \Delta V^2]^{1/2}$, where $\Delta Q = Q' - Q$, etc.) divided by the total intensity, I, is

$$\delta\pi \equiv \frac{\Delta I_{pol}}{I} = \left[(\Delta G/2)^2 + (\gamma\sin\phi_\gamma)^2 + (1 - \gamma\cos\phi_\gamma)^2\right]^{1/2}. \tag{3}$$

Defining a gain ratio $R = G_y/G_x$ and setting $G_x = 1$, the polarization error is

$$\delta\pi = \frac{1}{2}\left[(1+R)^2 - 8\sqrt{R}\cos\phi_\gamma + 4\right]^{1/2}. \tag{4}$$

For a pure phase error, the polarization error reduces to $\delta\pi = 2|\sin\phi_\gamma/2|$.

Polarization Isolation

Define the degree of isolation in terms of the (pseudo) LHCP that is measured in response to the incident RHCP:

$$i \equiv \frac{I'_L}{I'} = \frac{I' + V'}{2I'}. \tag{5}$$

Substituting the gain ratio $R \equiv G_y/G_x$, the isolation becomes

$$i = \frac{1 + R - 2\sqrt{R}\cos\phi_\gamma}{2(1+R)}. \tag{6}$$

For a 1 dB amplitude mismatch and no phase error, Eq. 6 gives $i = 0.0033$ or -24.8 dB of isolation. However, the errors in Stokes parameters are not so small: Eq. 4 indicates that the polarized flux is in error by $\delta\pi \sim 22\%$. Each Stokes parameter is in error by an amount $\sim \delta\pi/\sqrt{3}$. Scientifically useful results require precisions in Stokes parameters better than 1%. It does not appear practial to strive for this precision in hardware because other effects, such as cross coupling in the feed, require correction in software using a variety of calibration measurements using celestial sources.

Figures

Figure 1 shows the polarization error $\delta\pi$ plotted against the gain ratio R for different values of phase error, ϕ_γ. Note that as R gets larger, the polarization error becomces increasingly less sensitive to phase errors. Also, for realistically achievable gain and phase mismatches, the polarization error does not get better than -15 dB.

Figure 2 shows contours of constant$i$ against phase error (ϕ_γ and the amplitude ratio, R.

What Level of Instrumental Polarization is Tolerable?

Examples given above suggest that better than 25 dB of isolation is needed. However, even -30.8 dB isolation (or R of 0.5 dB) still gives 6% errors in individual Stokes parameters. The only way to achieve 1% errors is to correct Stokes parameters in software by inverting Eq. 1. More accurately, one needs to invert a similar matrix (given in our 1992 memo) that includes cross coupling in the horn + OMT as well as hyrbrid errors.

A workable situation may be defined in terms of our experience with the line feed antennas. There we had a level of cross coupling that gave about 10% errors in Stokes parameters. Through tracking of polarized sources across parallactic angle, the cross coupling parameters could be determined and the Stokes parameters corrected to about 1%.

With the more complicated situation involving the hybrid combined with cross coupling, the hardware should be designed to yield $\lessapprox$ 10% errors in Stokes parameters. This corresponds to an amplitude ratio of 0.8 dB or less ($0.8 \lessapprox R = G_y/G_x \lessapprox 1.2$) or a phase error $|\phi_\gamma| \lessapprox 0.1\,\mathrm{rad} = 5.^\circ 7$, corresponding to an isolation $i \lessapprox -26$ dB. If this can be achieved, then the remaining correction can be made in software.

How bad can the instrumental polarization be and still allow inversion of the matrix relation? The bigger the off-diagonal terms in the 4×4 matrix, the more unstable the correction and the more accurately that the matrix elements need to be determined. For example, inversion of the 2×2 submatrix involving I, Q in Eq. 1 involves the determinant $D = \overline{G}^2 - (\Delta G/2)^2$ in the denominator of the matrix elements. The elements become unwieldy as $\Delta G/2\overline{G} \to 1$.