CORRELATOR POLARIMETRY

Jim Cordes

1995 November 27

This writeup is intended to clarify the relationship between polarimetry done with a correlator and continuum polarimetery done with a single-channel system. First I derive the correlation functions for a monochromatic signal and then a continuum signal. These show explicitly how the Q and U Stokes parameters are related to the symmetric and antisymmetric components of the cross-correlation function of the LHCP and RHCP voltages.

Monochromatic, Linearly Polarized Signals

Consider a monochromatic, linearly polarized signal at RF:

$$\vec{E}(t) = c\cos(\omega t)\hat{x} + s\cos(\omega t)\hat{y}. \tag{1}$$

where $c = \cos\chi$ and $s = \sin\chi$ and χ is the position angle. The corresponding components of circular polarization are

$$E_{L,R}(t) = E_x(t) \pm E_y(t - t_{1/4}) \tag{2}$$

where $t_{1/4} \equiv \pi/2\omega_0$ is a delay equal to one quarter cycle at the center frequency ω_0. The IF signals are (with implicit filtering off of the upper sideband)

$$L_{IF}, R_{IF} = E_{L,R}(t)\cos(\omega_{LO_1} t) \tag{3}$$

and the baseband voltages are (again with implicit filtering)

$$\ell(t), r(t) = L_{IF}, R_{IF}(t)\cos(\omega_{LO_2} t). \tag{4}$$

The autocorrelation functions (acfs) of the baseband fields are

$$\langle \ell(t)\ell(t+\tau)\rangle = \frac{1}{2}\cos(\delta\omega\tau)[c^2 + s^2 + 2cs\cos(\omega t_{1/4})] = \frac{1}{2}\cos(\delta\omega\tau) \tag{5}$$

$$\langle r(t)r(t+\tau)\rangle = \frac{1}{2}\cos(\delta\omega\tau)[c^2 + s^2 - 2cs\cos(\omega t_{1/4})] = \frac{1}{2}\cos(\delta\omega\tau) \tag{6}$$

from which two of the Stokes parameter correlations may be defined:

$$\begin{aligned} I(\tau) &= \langle \ell(t)\ell(t+\tau)\rangle + \langle r(t)r(t+\tau)\rangle = \cos(\delta\omega\tau) \\ V(\tau) &= \langle \ell(t)\ell(t+\tau)\rangle - \langle r(t)r(t+\tau)\rangle = 0. \end{aligned} \tag{7}$$

The cross correlation function (ccf) between ℓ and r yields $L(\tau) \equiv Q(\tau) + U(\tau)$:

$$\begin{aligned} L(\tau) = 2\langle \ell(t)r(t+\tau)\rangle &= [(c^2 - s^2)\cos(\delta\omega\tau) - 2cs\sin(\delta\omega\tau)\sin(\omega t_{1/4})], \\ &= \cos 2\chi\cos\delta\omega\tau - \sin 2\chi\sin\delta\omega\tau \\ &= \cos(\delta\omega\tau + 2\chi) \end{aligned} \tag{8}$$

where $\delta\omega \equiv \omega - \omega_{LO_1} - \omega_{LO_2}$ is the baseband frequency of the monochromatic signal. Lines 1-2 of Eq. (8) explicitly show symmetric and antisymmetric parts of $L(\tau)$. When transformed to the frequency domain, they correspond to $Q(\tilde{\omega})$ and $U(\tilde{\omega})$, respectively. The position angle is found in the usual way to be

$$\chi_{\tilde{\omega}} = -\left(\frac{1}{2}\right) \tan^{-1} \frac{U(\tilde{\omega})}{Q(\tilde{\omega}).} \tag{9}$$

Linearly Polarized Continuum Signal

Now consider a continuum signal, again with 100% polarization, analyzed in a total bandwidth B. We assume there is no Faraday rotation across the bandwidth (see below). Stokes parameters may be found by integrating the monochromatic result over frequency because the SP's are variance-like quantities and the frequency components are statistically independent (variances add). Performing integrals like

$$\begin{aligned} I(\tau) &= \int_0^B d\delta\omega I_m(\tau) \\ L(\tau) &= \int_0^B d\delta\omega L_m(\tau), \end{aligned} \tag{10}$$

where I_m, L_m are the monochromatic results from Eq. (7)-(8) [that depend on the frequency $\delta\omega$], we find that

$$\begin{aligned} I(\tau) &= \frac{\sin B\tau}{\tau} \\ L(\tau) &= \cos 2\chi \left(\frac{\sin B\tau}{\tau}\right) + \sin 2\chi \left(\frac{\cos B\tau - 1}{\tau}\right) \\ V(\tau) &= 0. \end{aligned} \tag{11}$$

When transformed to the frequency domain, Eq. (11) yield the Stokes parameters vs. frequency. In this case, the SPs are independent of frequency and it may be seen that a linearly polarized continuum source with arbitrary position angle is representable with correlation functions as we have defined them in Eq. (5) - (8).

Elliptically Polarized Signals

A monochromatic signal with arbitrary elliptical polarization is handled as above. General expressions that include differential timing delays, Faraday rotation, and LO phase offsets may be found in Eq. (C3)-(C4) in my 1988 memo, *Polarimetry with the 40 MHz Correlator*. Similarly, noise-like signals with arbitrary polarization may be analyzed by integration of the monochromatic results, as we did above.

Single Channel Polarimetry

By single channel polarimetry, I mean a system where only a single lag of the auto-and-cross correlations is computed. This might be effected with analog multipliers rather than a digital correlator. In this case, one must explicitly calculate the ccf between the RHCP and LHCP components *with* a 90° phase shift as well as without a phase shift. This may be seen by using Eq. (1)-(2) and calculating the cross correlations (for a monochromatic signal)

$$2\langle E_R(t)E_L(t+\tau)\rangle = \cos 2\chi \cos \omega\tau + \sin 2\chi \sin \omega\tau \tag{12}$$

$$2\langle E_R(t)E_L(t+\tau-t_{1/4})\rangle = \cos 2\chi \sin \omega\tau - \sin 2\chi \cos \omega\tau. \tag{13}$$

At zero lag ($\tau = 0$), the unshifted correlation (Eq. 12) gives $\cos 2\chi$ while the shifted correlation (Eq. 13) gives $\sin 2\chi$; both $\cos 2\chi$ and $\sin 2\chi$ are needed to solve for the position angle without ambiguity.