Arecibo Upgrade Notes

INSTRUMENTAL POLARIZATION: CROSS COUPLING AND HYBRID CONVERSION ISSUES

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Summary

The standard Gregorian feed and receiver system will yield linear polarizations at the output of the orthomode transducer (OMT) and after at least the first stage of amplification. Many scientific applications, including VLBI, pulsar, and Zeeman-splitting observations require routine measurement of circular polarizations. The linears thus need conversion to circular polarization as close to the RF stage as possible (to minimize the number of active elements whose gains may vary with time). Here, we consider the influence of gains and phase shifts on the quality of the resultant polarization. We also outline schemes for calibrating the polarization of large bandwidth systems. We consider two cases: (A) When cross coupling in the feed and OMT can be ignored, instrumental calibration is accomplished through measurements on (1) an unpolarized source; (2) a linearly polarized source; and (3) noise injected into the OMT. In this case, it is desireable that the relative x,y gain and phase be as close to ideal as possible; the remaining small imperfections can and must be calibrated out to achieve 1% polarization. (B) We also consider cross coupling combined with imperfect conversion. Calibration of this system is more difficult but feasible. The additional two calibration parameters require tracking of a linearly polarized source through a large range of parallactic angle or, equivalently, through 180° of mechanical rotation. However, this needs doing only once because the horn + OMT combination is expected to be very stable. Occasional monitoring (once per month) may be warranted, however, to check the dependence of cross coupling on the positioning of the Gregorian system. The savings in antenna time provided by mechanical rotatability of the feed are substantial and we strongly recommend that as many of the feeds be rotatable as are allowed by financial and weight limits.

Signal Model

Let $\hat{x}$ and $\hat{y}$ denote unit vectors in the two directions of linear polarization selected by the feed + OMT combination. We assume that the linear signals from the OMT are perfect. In practice, they are nonorthogonal and there is cross coupling; we consider cross coupling later. Here, we are concerned with conversion to circular. Total gains from primary through gregorian optics, feed, OMT, inital LNA, and paths in a quadrature hybrid are $g_{x,y}$. Here and in the following, all such gains are recognized to be functions of frequency, ω. The hybrid combines the x, y signals into effective unit vectors for nominal (primed) circular polarizations:

$$\hat{\epsilon}'_{R,L} = \frac{1}{\sqrt{2}}\left[g_x(\omega)\hat{x} \pm g_y(\omega)e^{i\phi(\omega)}\hat{y}\right], \tag{1}$$

where $\phi(\omega)$ is the phase shift imposed by the hybrid. Perfect conversion requires $g_x = g_y = 1$ and $\phi = \pi/2$. Writing the phase in terms of its deviation from $\pi/2$, or $\phi = \pi/2 + \delta\phi(\omega)$, we have

$$\hat{\epsilon}'_{R,L} = \frac{1}{\sqrt{2}} \left[g_x(\omega)\hat{x} \pm i g_y(\omega) e^{i\delta\phi(\omega)} \hat{y} \right]. \tag{2}$$

Given the unit vectors, the nominal right-and-left-hand circular polarizations may be written in terms of the (filtered) electric field $\vec{E}$ (using '*' to denote complex conjugate)

$$E'_{R,L} = \vec{E} \cdot \hat{\epsilon}'^{*}_{R,L}. \tag{3}$$

Measured voltages at the hybrid output $V'_{R,L} = g_{R,L} E'_{R,L}$ involve gains $g_{R,L}$ whose determination we assume is perfect.

Stokes Parameters

An elliptically polarized signal may be written generally in terms of perfect circular unit vectors $\hat{\epsilon}_{R,L}$

$$\vec{E} \equiv E_R\, \hat{\epsilon}_R + E_L\, \hat{\epsilon}_L. \tag{4}$$

The Stokes parameters I,V are calculated from the sum and differences of the magnitudes of the nominal (primed) circular electric fields; the Stokes parameters Q, U derive from the cross correlation of the electric fields. By necessity, Stokes parameters are calculated from differential on-and-off source measurements.† For steady sources, the on−off measurements are made through true position switching whereas pulsars provide on-and-off pulse time spans without moving the telescope. The estimated Stokes parameters (primed) are related to the true (unprimed) Stokes parameters as

$$\begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} = \begin{pmatrix} \overline{G} & \frac{\Delta G}{2} & 0 & 0 \\ \frac{\Delta G}{2} & \overline{G} & 0 & 0 \\ 0 & 0 & \gamma \cos\phi_\gamma & -\gamma \sin\phi_\gamma \\ 0 & 0 & \gamma \sin\phi_\gamma & +\gamma\cos\phi_\gamma \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}, \tag{5}$$

where

$$\overline{G} = \frac{1}{2}(G_x + G_y) \tag{6}$$

† Note that unpolarized system noise in the x, y linear signals will yield Stokes parameters with $V = U = 0$ but with $Q \neq 0$ according to the differences in system noise and in the x, y power gains; ie. $Q = G_x N_x - G_y N_y$ where $N_{x,y}$ is the noise power and $G_{x,y}$ is defined in eqn (7). Differencing on-and-off source measurements will remove this pseudo linear polarization.

$$G_{x,y} = |g_{x,y}|^2 \tag{7}$$

$$\Delta G = G_x - G_y \tag{8}$$

$$\gamma = (G_x G_y)^{1/2} \tag{9}$$

$$\phi_\gamma = \phi_x - \phi_y - \delta\phi, \tag{10}$$

$\phi_{x,y}$ is the phase of the complex gain $g_{x,y}$ and $\delta\phi$, as before, is the phase error in the hybrid. Inspection of this matrix equation shows that gain and phase errors couple I with Q and U with V.

Data sheets on octave-bandwidth quadrature hybrids indicate that the gain difference is about 0.5 dB and the phase balance is about $\pm 2°$. These numbers imply errors in Stokes parameters of 5 to 10%. However, additional contributions to the gains and phase arise from active components before the hybrid and it is likely that imperfections in these components will be worse than in the hybrid. Precision polarization work will therefore require removal of instrumental polarization.

Calibration Schemes

Removal of instrumental polarization due to these effects requires determination of the three quantities G_x, G_y, and ϕ_γ. To do so, a series of measurements is needed on unpolarized sources, linearly polarized sources, and noise injected into the OMT. The gains and phases will be time variable owing to temperature variations, changes in bias voltages, etc. However, the time variable components of the gain are likely to be in the OMT onward while those in front of the OMT are time independent (apart from pointing variations or, more drastically, wind damage to the primary reflector or components of the Gregorian optics!) Thus it probably will suffice to rely on a combination of infrequent measurements that first determine the total gains applied to signals from an astronomical source and frequent measurements on injected noise that measure gains from the OMT onward.

In the following, we drop explicit frequency dependence. It should be assumed, however, that all quantities are evaluated over narrow bandwidths provided by a channelization device, such as a correlator or FFT based system.

Unpolarized Source: $Q = U = V = 0$, so the only nonzero *measured* Stokes parameters are

$$I' = \frac{1}{2}(G_x + G_y)I \tag{11}$$

$$Q' = \frac{1}{2}(G_x - G_y)I. \tag{12}$$

Measurement of these allows solution of $G_{x,y}$ as a function of ω over the entire band of interest.

Linearly Polarized Source: $V = 0$. Using solutions for $G_{x,y}$ from measurements of an unpolarized source, the true I and Q can be solved for. Measurements of $U' = \gamma \cos \phi_\gamma U$ and $V' = \gamma \sin \phi_\gamma U$ yield a solution

$$\phi_\gamma = \tan^{-1} \frac{V'}{U'} \tag{13}$$

and, hence, solutions for the true U and V. A variant on this is measurement of injected noise. If such noise is unequal and uncorrrelated between the x, y channels, the implied Stokes parameters are like those of a linearly polarized signal. Thus, frequent measurement of the Stokes parameters for injected noise allows monitoring of time variations in $G_{x,y}$ and in the net phase difference, ϕ_γ. The cost of these measurements in telescope time is small compared to that of, say, extragalactic sources.

Rotating Feed: Without a compensating mechanism, the feed will rotate with parallactic angle. With a mechanism, the rotation can be removed or increased in order to diagnose instrumental polarization. The Q, U parameters will vary with such rotation according to a rotation matrix having $\cos 2\chi_f$ and $\sin 2\chi_f$ as elements, with χ_f being the position angle of the feed antenna. With such rotation, inspection of eqn (5) shows that each of U', V' consist of a constant term (proportional to the true V) and a variable term proportional to U. By making a sequence of measurements on an elliptically polarized source covering a large range of χ_f, one may solve for the variable and constant parts and solve for ϕ_γ and γ.

Cross Coupling

The analysis above assumes the x and y signals to be perfectly linearly polarized. In practice, these signals will be cross coupled according to a unitary matrix relating perfect (unprimed) to imperfect (primed) signals:

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = \begin{pmatrix} \sqrt{1-\epsilon} & -\sqrt{\epsilon} e^{i\psi} \\ +\sqrt{\epsilon} e^{-i\psi} & \sqrt{1-\epsilon} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}, \tag{14}$$

where ϵ and ψ are the amplitude and phase of the cross coupling. The transformation conserves polarized power and corresponds to the case where the nominal x, y signals are elliptically but orthogonally polarized, a model that works well for the line feed antennas at Arecibo. In fact, this model has failed only during an instance when one of the line feeds became severely mispointed owing to improper installation after hanger replacement in 1992 February.

Cross coupling combined with differential gains and phases in the hybrid yield a transformation matrix:

$$\begin{pmatrix} \overline{G} & \frac{\Delta G}{2}(1-2\epsilon) & -\Delta G\eta\cos\psi & -2\overline{G}\eta\sin\psi \\ \frac{\Delta G}{2} & \overline{G}(1-2\epsilon) & \overline{G}\eta\cos\psi & -\Delta G\eta\sin\psi \\ 2\gamma\eta\sin\psi\sin\phi_\gamma & 2\gamma\eta\cos\psi\cos\phi_\gamma & \gamma\cos\phi_\gamma(1-2\epsilon) & -\gamma\sin\phi_\gamma \\ -2\gamma\eta\sin\psi\cos\phi_\gamma & 2\gamma\eta\cos\psi\sin\phi_\gamma & \gamma\sin\phi_\gamma(1-2\epsilon) & +\gamma\cos\phi_\gamma \end{pmatrix}, \tag{15}$$

where $\eta = [\epsilon(1-\epsilon)]^{1/2}$. With no cross coupling ($\epsilon = 0$), the matrix reduces to eqn (5).

Cross coupling in Arecibo's circularly polarized line-feeds implies errors in V of $2\sqrt{\epsilon} \sim 10\%$ of the linearly polarized power. Such errors are correctible if the parameters ϵ, ψ are determined through observations of linearly polarized sources. Feeds and OMT's for the Gregorian system may have significantly smaller cross coupling, but it is unlikely that ϵ will be small enough to reduce errors below 1%. Precision of 1% requires $\epsilon < 2.5 \times 10^{-5}$, or coupling that is $\times 100$ smaller than for line feeds. Consequently, precise polarization measurements require correction for cross coupling.

Calibration of the Gregorian feed systems against cross coupling requires determination of an additional two parameters, for a total of five. Inspection of eqn (15) shows that observations of an unpolarized source allow solutions for the first column, including those for $G_{x,y}$, as before with no cross coupling, from measurements of I' and Q'. With cross coupling, U' and V' are also nonzero and allow solution for $\eta\sin\psi$ and ϕ_γ. If one assumes further that $\epsilon \ll 1$ but $\sqrt{\epsilon}$ cannot be ignored, then most matrix elements can be determined from observations of an unpolarized source, to a precision of $1 - 2\epsilon \approx 1$. However, elements involving $\cos\psi$ are not determinable from single measuements of a linearly polarized source. Instead, measurements over 180° of feed rotation are needed. As above, a linearly polarized source will show Q, U varying as $\cos 2\chi_f$ and $\sin 2\chi_f$, with χ_f being the orientation angle of the feed. All four measured Stokes parameters contain terms that vary with χ_f added to terms that remain constant. Those variable terms containing $\cos\psi$ can be used to solve for $\sqrt{\epsilon}\cos\psi$. As an illustration of this process, we show in Fig. 1 the measured and "true" Stokes parameters as a function of parallactic angle for a linearly polarized source.

Conclusions

A general model for instrumental polarization has been used to outline methods for determining the relevant instrumental parameters that are needed for removal of instrumental effects. Some of these parameters are expected to be time dependent, so rather frequent calibration data must be taken (eg more than once per hour). Other parameters, particularly those describing cross coupling in the feed, are expected to be steady in time. However, *all* parameters are expected to be functions of frequency, so the calibration scheme must be applied with sufficient frequency resolution. Use of a correlator to obtain auto-and-cross correlation functions of the two receiver signals is ideal for this purpose. As will be detailed elsewhere, the precision of sampler thresholds prior to correlation must be sufficient to allow solutions for gains to better than 1%.

References

Perley, R., Schwab, F., and Bridle, A. 1986, *Synthesis Imaging*, NRAO.

Stinebring, D.R., Cordes, J.M., Weisberg, J.M., Rankin, J.M., and Boriakoff, V. 1984, *ApJ*, **55**, 247.

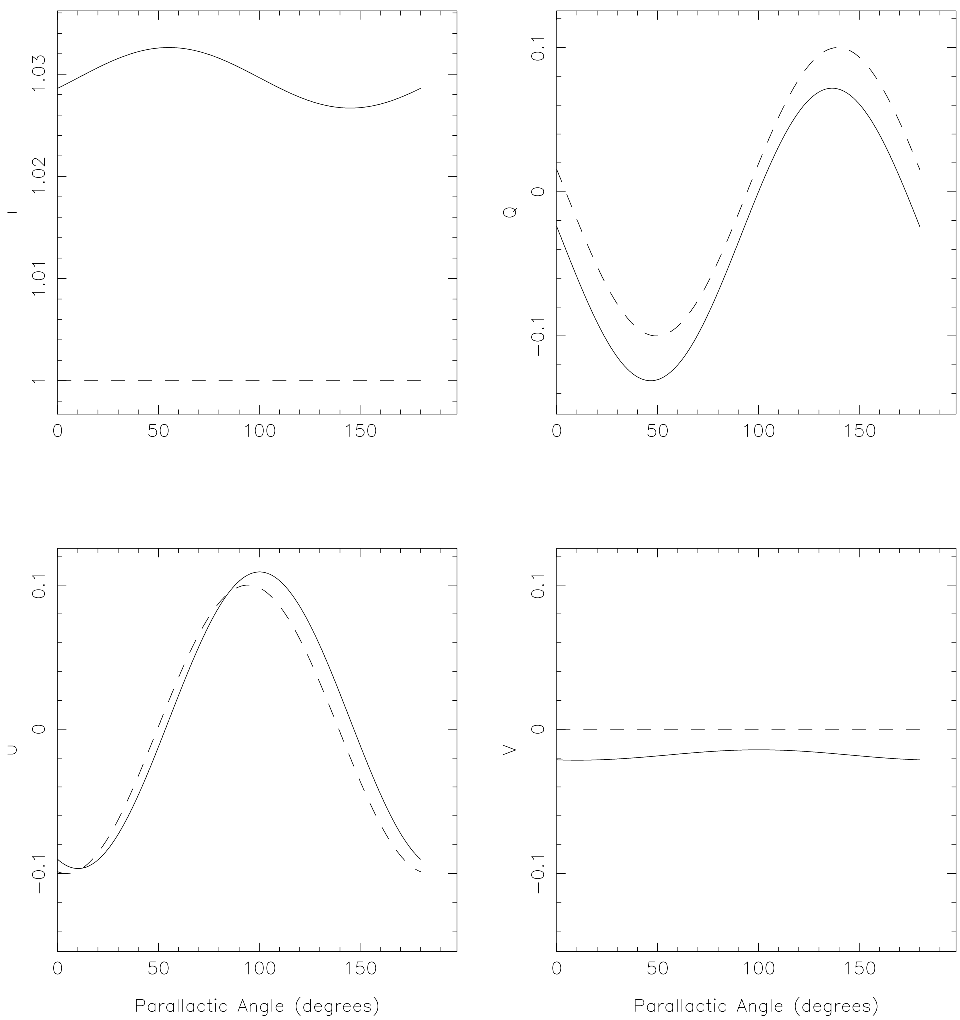

Fig. 1. Illustration of the effects of cross-coupling and gain difference. Dashed lines are the "true" Stokes parameters, i.e. those observed with an ideal antenna and with perfect gain calibration, for which $\bar{G} = 1$ and all other parameters are zero. The feed rotates with parallactic angle, however, giving rise to sinusoidal variations in Q and U. Solid lines depict the results when cross coupling and relative gains are not ideal. The source is assumed to be 10% linearly polarized with no circular polarization. A nominal gain difference of 0.5 dB and phase of 2° with a cross coupling amplitude of 1% and phase of 5° were assumed. Because all of these quantities will vary with frequency, the dependences on parallactic angle will also vary with frequency. Calibration must therefore be carried out for individual frequency channels.