POLARIMETRY WITH THE 40 MHz CORRELATOR

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I. INTRODUCTION

A method has been developed for using the 40 MHz correlator as a dedispersing, multiplying polarimeter of pulsar signals. Here I briefly describe the performance of the hardware and software. Examples of Stokes parameter waveforms are shown.

Polarimetry done heretofore at the Arecibo Observatory has mostly used one of the adding polarimeters to study pulsars with millisecond resolution (e.g. Backer and Rankin, *Ap. J. Suppl.*, 42, 143, 1980; Stinebring, Cordes, Rankin, Weisberg, and Boriakoff, *Ap. J. Suppl.*, 55, 247, 1984). Microsecond resolution requires predetection dispersion removal followed by multiplying polarimetry in the computer (Cordes and Hankins, *Ap. J.*, 218, 484, 1977; Stinebring and Cordes, *Nature*, 306, 349, 1983). Multiplying polarimetry with the new correlator was suggested by Kulkarni, Clifton, and Frail in their memorandom (1986) describing use of the correlator for pulsar HI absorption observations. The present report describes an implementation of correlator polarimetry.

Digital, multiplying polarimetry with the correlator is superior to use of an analog filter bank and adding polarimeter because: 1) the frequency resolution may be optimized to a given pulsar and observation frequency; 2) the net bandpass shape is stable with time; 3) the resultant Stokes parameters are far more robust to errors in calibration constants; and 4) broadband measurements may be made with impunity because no 90° phase shift, which would have to be constant over the observing bandwidth, is applied to the IF signals as in an adding polarimeter. Polarization data may be obtained with flexible numbers of frequency channels and resolution cells across the pulsar period, within the constraints specified in Computer Department Report # 23 by Phil Perillat. Millisecond pulsars probably require other techniques because the correlator is hardware limited in providing only modest ($\geq 300\mu$s) time resolution.

The system has been tested in a program aimed at monitoring the polarization waveforms of the 8 hour binary pulsar 1913+16 for the purpose of detecting geodetic precession of the spin axis. The main diagnostic is the polarization position angle signature, which requires accurately calibrated Q and U waveforms. In addition, the position angle must be referenced to some fixed angle on the sky. So far, we have used a nearby, highly linearly polarized pulsar (1929+10) as a position angle reference. In the future, we intend to use extragalactic radio sources as a grid for position angle references. In this way, Arecibo observations can be tied into polarization observations made with the VLA.

The tex file for this document is contained in [cordes.geodetic]stokes1.tex on the Arecibo VAX and the Cornell ASTROVAX.

II. BASIC PHILOSOPHY

The derivation of Stokes parameters from correlator measurements is described in Appendix A. Instrumental cross polarization is analyzed in Appendix B, while the effects of instrumental time and phase delays is treated in Appendix C. Here, we focus our attention on the logistical and computational details of correlator based polarimetry.

Data acquisition makes use of the program XCORHI, which allows correlation functions to be obtained for individual pulse phases while averaging over many pulse periods. XCORHI and associated jargon are described thoroughly in Computer Department Report #23 by Phil Perrilat. In the following, variable names relevant to XCORHI input files will be used.

XCORHI uses dump mode 0 ('radar mode' as in Figure 3.4 of *INSTRUCTION MANUAL, Project No. 579, 40 MHz Correlator*, Manual No. 8319 of the Arecibo Observatory Electronics Department). Briefly, XCORHI accumulates correlation functions with LENSBC lags each in a specified integration time (DUMPT) and dumps them to contiguous segments of correlator memory. There is a 6μs dead time between the end of one integration and the beginning of another. Once 16k words of correlations have been obtained, they are shipped to the Harris H800 and then to the array processor. Data transfer to the Harris and to the AP is double buffered. In the array processor, each dump is added into the appropriate pulse phase bin, of which there are a total of APBINS phase bins. The first dump of a scan goes to the first phase bin; later dumps are placed taking into account the dumptime, the 6μs dead time between dumps, the dead time for switching buffer memory in the the correlator (constrained to 40μs by using an external start signal, as in the diagram in Figure 2) and a current pulsar period (P).

After a scan has finished, correlations are written to tape. The data set on tape is 3 dimensional (correlation value as a function of lag, pulse phase, and subcorrelator number [4]). Data reduction consists of forming spectra, dedispersing, and calibration to form Stokes parameter waveforms, thereby collapsing the data into 2 dimensions: flux density as a function of pulse phase for each of 4 Stokes parameters. Extragalactic source data (taken with the same parameters as the pulsar data) are used to estimate calibration constants. The resultant waveforms have a net time resolution that is roughly the quadrature sum of DUMPT, (P/APBINS), and the dispersion delay across one frequency channel.

For polarimetry, XCORHI is run with 4 subcorrelators (NUMSBC 4) and cross correlation invoked (CROSS Y). Two signals (e.g. RHCP and LHCP) are used as inputs to baseband mixers 1 and 2. The resultant real correlations (LENSBC lags) of the real signals contain (with 'acf' = autocorrelation function and 'ccf' = cross correlation function):

subcorrelator 1 = acf of input 1

subcorrelator 2 = ccf of input 1 with input 2

subcorrelator 3 = acf of input 2

subcorrelator 4 = ccf of input 2 with input 1.

The Fourier transforms of subcorrelators 1 and 3 are proportional to the intensities of the two input signals in each of LENSBC frequency channels. Subcorrelators 2 and 4 are combined as, respectively, the + and − lags of a joint cross correlation function (ccf). The Fourier transform is proportional to $Q + iU$ versus frequency. Thus the symmetric part of the joint ccf determines Q while the antisymmetric part determines U. Note that the FFT of the joint ccf is of length 2×LENSBC while subcorrelators 2 and 4 provide only 2×LENSBC−1 *unique* values because the zero lag value is redundant. Software described below fills in the missing LENSBC + 1^{th} element by averaging the two adjacent values. More details are given in Appendix A.

To dedisperse, the data are summed over frequency while imposing time delays in accord with the cold plasma dispersion law rather than a linear law, as is sometimes used. No doppler shift is taken into account in the dispersion removal, however. Before summing the Q and U waveforms across frequency, they must be derotated in the Q, U plane. Rotations occur because of Faraday rotation in the interstellar medium and ionosphere and, more importantly, because of instrumental time delays between the two input channels to the correlator which mimic Faraday rotation. For most objects, true Faraday rotation is unlikely to be large across feasible bandwidths. A famous incident of pseudo Faraday rotation occurred in early 1988, however, when (unknown to us) a 350 ft length of cable was replaced in one of the cable paths from carriage house 2. Although nominally the same length and type as the cable it replaced, the dielectric constant was about 10% different, enough to cause $\sim$ 40 ns propagation time delay, enough to completely quench the linear polarization across a 20 MHz bandpass. After this incident, the analysis programs were modified to derotate Q and U.

The system has been used with the following parameters: 20 MHz total bandwidth, 32 frequency channels (LENSBC), 128 or 256 pulse phase bins (APBINS), and dumptimes (DUMPT) of 400μs or larger.

Analysis programs address the following aspects of calibration:

1. Calibration of relative intensities using the zero lag values of the correlation functions and knowledge of the digital threshold used in the 3 level sampling. The digital threshold is constant during a scan but changes from scan to scan.

2. Bandpass calibration of all elements in the system from the feed antenna to the base-band mixer filters.

3. Relative gains through the system between the two (e.g. RHCP and LHCP) input channels and absolute calibration with respect to an extragalactic point source of known flux density.

4. Correction for the zenith angle dependence of the antenna gains.

5. Correction for parallactic angle rotation of the feed.

6. Determination of feed cross coupling parameters and removal of cross coupling effects from the V waveforms. Details and test results are given in Appendix B.

7. Determination and correction for rotation of Q and U between frequency channels caused (primarily) by unequal propagation times of the two receiver signals.

III. AVAILABLE PROGRAMS

The following describes programs used to acquire, analyze, and display polarization waveforms. Except where noted, all programs reside on the Harris H1000 computer at the Arecibo Observatory (in the 110PULS area).

A. DATA ACQUISITION as described in Computer Department Report #23.

   1. DOPSY or ORBDOP to generate a doppler shift file PULSEP. ORBDOP handles binary pulsars if orbital data is properly put into the pulsar data file.

   2. SWIFT to transfer the file PULSEP from the H1000 computer to the H800 computer.

   3. WINDOW to drive the SPS for triggering of an oscilloscope or the signal averager and for driving the noise calibration signal in the receiver front end (H800). I usually do not use a noise pulse, since the data are calibrated against an extragalactic source.

   4. XCORHI to acquire correlation functions averaged over pulse phase (H800). A sample input file is shown in Figure 1. It resides in 110PULS*P684POL on the H800 computer. A setup for 1.4 GHz appears in Figure 2.

   5. Accurate polarimetry requires use of external LO's for the baseband mixers. The *same* external synthesizer signal should be used for both mixers. Internal LO's are accurate to only a few parts in 10^6 ($\sim$ 1 kHz). Their use reduces the cross correlations, thereby depolarizing any linear polarized component.

   6. Determination of calibration constants requires on source and off source scans on a point extragalactic source with negligible circular polarization. The observing setup must be the same as that used for pulsar measurements. I usually do a triplet consisting of north by 10 arc min, on source, and south by 10 arc min for frequencies of 1.4 GHz and higher.

B. CALIBRATION

   1. PCAL reads scans made on and off an extragalactic radio source. The program prompts for an input file (see example in file PCALIN) which contains the tape

id, calibration source name and flux density, and scan numbers to be processed. Each scan number is labelled as on or off source and analyzed accordingly. Proper correction is made for the zenith angle dependence of the gain.

2. Output files (all ASCII) are PCALDIAG (a dump file with considerable diagnostic information), PCALSUMM (a summary file of the calibration processing), BPASS (the bandpass shapes of each subcorrelator; see example in Figure 3), and PCALOUT (the calibration constants used for further processing).

C. STOKES PARAMETERS OF PULSAR DATA

1. The program CORPOL reads calibration constants from an input file (e.g. a copy of PCALOUT), reads pulsar data from tape, and calculates Stokes parameter waveforms in Jansky units. Correction is made for the zenith angle dependence of the antenna gain and Q and U are rotated to take out parallactic angle rotation of the feed. Frequency channels are summed after removal of dispersion delays of the lower frequency channels with respect to the highest frequency channel. Dispersion delays are removed in (floating point) units of the sample interval by computing the forward DFT, modifying the phases, and inverse transforming.

2. Input files are a prompted for file (see model in CORPOLIN) which contains the AO tape id, pulsar name, and scan numbers to process; PCALOUT (or a renamed version) with calibration constants from program PCAL. The name of the cal constant file is requested by the program.

3. Output files (all ASCII) are POLDIAG = diagnostic output, POLSUMM = brief summary file, and POLWAVE = output Stokes parameter waveforms. POLWAVE is a contiguous file containing waveforms from the different scans specified in CORPOLIN. Summing of waveforms to produce grand averages is done in the plotting program discussed below.

4. The program computes the Stokes parameter $V = I_1 - I_2$, where $I_{1,2}$ are the intensities in the first and second input channels. For the setup given in the example below, channel 1 corresponds to RHCP coming from the sky, so $V = I_R - I_L$.

D. WAVEFORM PLOTTING ON THE VAX750

1. VAXPLOT on the VAX750 (in [cordes.geodetic]) reads a user specified data file (e.g. a renamed output file POLWAVE from program CORPOL that has been transferred from the H1000 using the FTP utility) and plots the Stokes parameter waveforms to a metacode file. The metacode file is translated by using the system program PLOT. VAXPLOT on the AOVAX is a copy of the official version on the VAX750 at Cornell. I will update the AOVAX version as I make changes to the Cornell version.

2. At present, VAXPLOT can plot individual scans and grand average scans. The average is written to the file WAVEAVE.DAT. To construct averages, it aligns waveforms in pulse phase before summing. When averaging data from different days (which necessarily involves different observing setups), the program empirically rotates all scans onto the same position angle origin. It does so by applying a rotation to Q and U of a given scan that maximizes the average (over pulse phase) of the product $QQ_{ref} + UU_{ref}$, where 'ref' refers to a reference scan. Note that Q and U waveforms are averaged, not the position angle waveform.

3. VAXPLOT, if requested, will calculate cross coupling constants by performing a least squares fit to the quantity V/L as a function of parallactic angle or position angle. It will also remove the effects of cross coupling from the V waveforms, if requested. See discussion in Appendix B.

4. A summary file (VAXPLOT.LOG) lists the results of each scan, including the period averaged flux density. If a cross coupling fit is done, the input data to the fit are also written to this file. Cross coupling constants are written to CROSS_PARAMS.DAT.

E. WAVEFORM PLOTTING ON THE HARRIS H1000

1. The program VAXPLT is a Harris version of the program VAXPLOT described above, with a few differences. The major one is that it plots to the screen or laser printer directly, rather than via a metacode translator that is run after the fact. This program is spawned from the original program on the Cornell VAX750. As such, it will be updated less frequently than the VAX versions. The output files are WAVEAVE, VAXPLLOG and CRSSPARM, corresponding to the VAX files WAVEAVE.DAT, VAXPLOT.LOG and CROSS_ PARAMS.DAT described above.

IV. EXAMPLES

Figure 1 shows a sample observe file that was used to drive the data acquistion program XCORHI. The observing setup is shown in Figure 2 for 21 cm observations. Figure 3 shows representative plots of the calibration constants versus frequency for the two input channels. Polarization waveforms of the strong pulsar 1929+10 are shown in Figure 4. Note that the position angle is nonrandom over the entire pulse period. This is a real effect and underscores the difficulty in defining a true 'off' region of pulse phase for some pulsars. Figure 5 shows waveforms for 0611+22 and Figure 6 shows waveforms for the 8 hour binary pulsar, 1913+16. Figure 7 shows the variation of the Stokes parameter V with position angle and a fit to determine the parameters of the underlying cross coupling. See Appendix B for more details.

V. FUTURE DEVELOPMENTS

1. Development of a program to analyze radio sources other than pulsars. For unpulsed sources, XCORHI can be used with an arbitrary folding period. On source and off source observations play the roles of on pulse and off pulse measurements of pulsars. It is anticipated that polarization position angles using different feed antennas (e.g. 430 MHz to 2.3 GHz) can be combined to yield rotation measures as small as a few rad m^{-2}.

VI. ACKNOWLEDGMENTS

In developing the polarization technique, I had useful conversations with Mike Blaskiewicz, Andy Clegg, Phil Perrilat, Joel Weisberg, and Alex Wolszczan. They also provided subroutines that were used in programs PCAL and CORPOL.

APPENDIX A

DERIVATION OF STOKES PARAMETERS FROM CORRELATION FUNCTIONS

Let $R_{IF}(t)$ and $L_{IF}(t)$ be the IF signals at the inputs to the correlator baseband mixers that correspond to the right and left hand circularly polarized antenna ports. These signals are real. Mixing to baseband with an LO signal of the form $\cos[2\pi(\nu_{IF} - \Delta\nu/2)t]$ (where $\Delta\nu$ is the bandwidth) and filtering to keep only the frequency components between 0 and $\Delta\nu$ yields baseband signals $R(t)$ and $L(t)$. These are also real signals. The correlator computes 2 real autocorrelation functions and 2 real cross correlation functions:

$$C_{RR}(\tau) = \langle R(t)R(t-\tau)\rangle$$

$$C_{LR}(\tau) = \langle R(t)L(t-\tau)\rangle$$

$$C_{LL}(\tau) = \langle L(t)L(t-\tau)\rangle$$

$$C_{RL}(\tau) = \langle L(t)R(t-\tau)\rangle$$

where angular brackets denote averaging over a specified time (of order 1 ms for pulsar signals). These are defined only for $0 \le \tau \le LENSBC - 1$, where LENSBC is the number of lags in each correlation function.

The Stokes parameters of the radiation field impingent on the primary reflector are wanted. Let $I_L(\nu)$ and $I_R(\nu)$ be the intensities of the right and left hand components of the incident radiation field and $Q(\nu)$ and $U(\nu)$ be the Stokes parameters that measure linearly polarized power. The net instrumental gain and frequency response may be lumped into frequency dependent quantities $G_R(\nu)$ and $G_L(\nu)$. It may be shown that:

$$DFT\left[C_{RR}(\tau) + C_{RR}(2\times LENSBC - \tau) + FIX\,\delta_{\tau,LENSBC}\right] = G_R(\nu)I_R(\nu) \qquad (A1)$$

$$DFT\left[C_{LL}(\tau) + C_{LL}(2\times LENSBC - \tau) + FIX\,\delta_{\tau,LENSBC}\right] = G_L(\nu)I_L(\nu) \qquad (A2)$$

$$\begin{aligned} &DFT\left[C_{LR}(\tau) + C_{RL}(2\times LENSBC - \tau) + FIX\,\delta_{\tau,LENSBC}\right] \\ &\quad = \frac{1}{2}\left[G_R(\nu)G_L(\nu)\right]^{1/2}\left[Q(\nu) + iU(\nu)\right] \end{aligned} \qquad (A3)$$

for $\tau = 0, 1, \ldots, 2\times LENSBC - 1$ and where DFT denotes Discrete Fourier transform of length $2\times LENSBC$. In equations (A1) and (A2), the correlation functions are symmetric and the Fourier transforms are real. In equation (A3), the cross correlation is asymmetric;

the transform of the symmetric part is proportional to Q and the antisymmetric part transforms to U. As mentioned in the main text, the missing $\tau = LENSBC$ correlation value is guessed at and entered into the transformed array. The guess (FIX, the coefficient of the Kronecker δ in the above equations) is set equal to the average of the adjacent $\tau = LENSBC \pm 1$ values.

Calibration consists of determining the quantities $G_{R,L}$. An extragalactic radio source, assumed to have zero circular polarization, readily yields these quantities in units of Janskys per computer unit when off source measurements are differenced from on source measurements. Linearly polarized sources may be used for calibration since the cross correlations are not used to determine the gain constants. Note, however, that cross coupling in the presence of linearly polarized signals yields biased calibration constants. However, for 10% voltage cross coupling (typical) and $< 10\%$ linear polarization, the calibration bias is less than 1%. Pulsar observations are calibrated by differencing off pulse values from all pulse phases, followed by multiplication of equations (A1) - (A3) by the reciprocals of $G_R(\nu)$, $G_L(\nu)$, and $\frac{1}{2}[G_R(\nu)G_L(\nu)]^{1/2}$, respectively.

APPENDIX B

INSTRUMENTAL POLARIZATION: REMOVAL OF CROSS POLARIZATION EFFECTS

Cross coupling in the feed antenna mixes the Stokes parameters. That is, cross coupling is described by a 4×4 matrix that transforms true parameters into the measured ones. The simplest case involves feed polarizations that are orthogonal ellipses rather than circles. References include Rankin, Campbell, and Spangler (NAIC Report 46, 1976); Stinebring *et al.* (1984; *Ap. J. Suppl.*; 55, 247, 1984); NRAO Synthesis Mapping Workshop Proceedings (1986). Assuming proper gain calibration, the cross coupling comprises a unitary matrix that relates the measured electric fields (primed) to the actual fields:

$$\begin{pmatrix} E_1' \\ \\ E_2' \end{pmatrix} = \begin{pmatrix} \sqrt{1-\epsilon} & -\sqrt{\epsilon}e^{i\psi} \\ \\ +\sqrt{\epsilon}e^{-i\psi} & \sqrt{1-\epsilon} \end{pmatrix} \begin{pmatrix} E_1 \\ \\ E_2 \end{pmatrix},$$

where $E_{1,2}$ are circularly polarized wavefields, and ϵ and ψ are the amplitude and phase of the cross coupling. For this case, polarized power is conserved: Q and U are converted into V and *vice versa*, so ${Q'}^2 + {U'}^2 + {V'}^2 = Q^2 + U^2 + V^2$.

Under the assumption of small cross coupling, $\sqrt{\epsilon} \ll 1$, the Stokes parameters are modified (to first order in $\sqrt{\epsilon}$) as

$$L' e^{2i\chi'} \approx L e^{2i\chi} + 2\sqrt{\epsilon}\, V e^{i\psi}$$

$$V' \approx V - 2\sqrt{\epsilon}\, L \cos(\psi - 2\chi)$$

where the true linear polarization is defined as

$$L \equiv Q + iU \equiv \exp^{2i\chi}.$$

For small circular polarization ($V/L \ll 1$), the variation in linear polarization is negligible, i.e. $L \approx L'$, and if linear polarization is large, the normalized circular polarization is

$$r \equiv V'/L' \approx V/L - 2\sqrt{\epsilon}\cos(\psi - 2\chi). \qquad (B1)$$

The Arecibo telescope has an alt-az feed mount on which the feed rotates as a source is tracked. Its projected orientation (onto the sky) is the parallactic angle χ_p, so the measured position angle may be expressed as $\chi = \chi_0 - \chi_p$, where χ_0 is the position angle characterizing the radiation field incident on the telescope. Thus the ratio $r(\chi_p)$ is a function of parallactic angle, the argument of the cosine in eqn(1) being $\psi - 2\chi = 2[\chi_p - (\chi_0 - \psi/2)]$.

The program VAXPLOT performs a least squares fit to the data of the function

$$r(\chi_p) = \alpha + a\cos[2(\chi_p - \chi_{p_0})]$$

with α, a, and χ_{p0} as parameters. The fitted parameters can be used to correct the V waveforms (only of data obtained with the same observing setup, since the angle χ_{p0} depends on the position angle origin, which depends on particular cable lengths). The corrected Stokes parameter V has the form

$$V(\chi_0, \chi_p) = V'(\chi_0, \chi_p) - a\cos[2(\chi_p - \chi_0 + \chi_{0\,fit} - \chi_{p0})],$$

where $\chi_{0\,fit}$ is the intrinsic position angle (defined as the measured position angle corrected for parallactic angle rotation) for the data used in the least squares fit.

Results in Figure 7 show the measured and fitted ratio $r(\chi_p)$ vs χ_p for measurements on PSR 1929+10 using the 1.4 GHz circular feed. The amplitude of the cross coupling agrees with that found by Stinebring *et al.* (1984) using the same feed and the same pulsar; their measurements were made in 1981. With a modest amount of parallactic angle coverage, the cross coupling parameters can be determined to better than 10% accuracy. A typical amplitude is $\sqrt{\epsilon} \approx 0.1$. The Stokes parameter V can then be corrected to better than 2%. As long as the *true* V/L is less than 10%, the linear polarization will be correct to better than 2%.

The fitting of V/L may be done with respect to *measured* position angle rather than with respect to parallactic angle. If there is no time dependent Faraday rotation, these two angles should track each other. However, variable Faraday rotation is likely at low frequencies and therefore fitting with respect to position angle should be preferred. At 1.4 GHz, test observations yield about the same quality of fit for the two approaches. The program VAXPLOT allows selection of either approach.

APPENDIX C

EFFECTS OF INSTRUMENTAL TIME DELAYS

The feed and receiver systems inevitably introduce differential time and phase delays on the signals owing to different cable lengths and signal propagation speeds. Phase delays that are constant across the receiver bandwidth are of no consequence, since they cause only a constant rotation of the plane of polarization. Time delays, however, mimic differential Faraday rotation, and can therefore depolarize the cross polarization. Successful observations with a bandwidth $\Delta\nu$ require that the group delay difference be no larger than $0.18/\Delta\nu$ in order that the depolarization be no more than 5%.

Consider a receiver system that mixes from radio frequency (RF) ω to an intermediate frequency ω_{IF} with a local oscillator ω_{LO_1} followed by mixing to baseband with a local oscillator ω_{LO_2}. The frequencies in a bandwidth $\Delta\omega$ centered on RF ω_0 extend from 0 to $\Delta\omega$ at baseband. We assume that the two receiver channels contain signals from right and left hand polarized feed ports. The LO signals for the two channels are assumed to be frequency locked but may have phase shifts $\phi_{L,R}$ introduced at ω_{LO_1} and $\psi_{L,R}$ at ω_{LO_2}. The time delays in the RF part of the receiver are $\alpha_{L,R}$ in the RF sections and $\beta_{L,R}$ in the IF sections. We ignore any delays at baseband because they are unlikely to produce significant phase shifts.

The electric field incident on the feed is

$$\vec{E}(t) = E_x(t)\hat{x} + E_y(t)\hat{y}$$

where $\hat{x}, \hat{y}$ are two orthogonal unit vectors and the RF voltages are

$$E_{L,R}(t) = E_x(t) \pm E_y(t - t_{1/4})$$

where $t_{1/4} \equiv \pi/2\omega_0$ is a delay equal to one quarter cycle at the center frequency ω_0. In the circular line feed antennas, this delay results from tuning of the feed to the proper polarization responses, assumed optimum at the center frequency. The IF signals are

$$L_{IF}, R_{IF} = E_{L,R}(t - \alpha_{L,R}) \cos(\omega_{LO_1} t + \phi_{L,R}) \qquad (C1)$$

and the baseband voltages are

$$\ell(t), r(t) = L_{IF}, R_{IF}(t - \beta_{L,R}) \cos(\omega_{LO_2} t + \psi_{L,R}). \qquad (C2)$$

A monochromatic, elliptically polarized signal has the form

$$\vec{E}(t) = A\cos(\omega t + \phi_A)\hat{x} + B\cos(\omega t + \phi_B)\hat{y}.$$

This wave field yields autocorrelations

$$\langle \ell(t)\ell(t+\tau)\rangle = \frac{1}{2}\cos(\delta\omega\tau)[A^2 + B^2 + 2AB\cos(\phi_B - \phi_A - \omega t_{1/4})]$$

$$\langle r(t)r(t+\tau)\rangle = \frac{1}{2}\cos(\delta\omega\tau)[A^2 + B^2 - 2AB\cos(\phi_B - \phi_A - \omega t_{1/4})]$$

from which two Stokes parameter correlations may be defined:

$$\begin{aligned} I(\tau) &= \langle \ell(t)\ell(t+\tau)\rangle + \langle r(t)r(t+\tau)\rangle = \cos(\delta\omega\tau)(A^2 + B^2) \\ V(\tau) &= \langle \ell(t)\ell(t+\tau)\rangle - \langle r(t)r(t+\tau)\rangle = 2AB\cos\delta\omega\tau)\cos(\phi_B - \phi_A - \omega t_{1/4}). \end{aligned} \quad (C3)$$

The cross correlation between ℓ and r is

$$\begin{aligned} L(\tau) = 2\langle \ell(t)r(t+\tau)\rangle &= [(A^2 - B^2)\cos(\delta\omega\tau + \omega\Delta\tau_{RF} + \omega_{IF}\Delta\tau_{IF} + \Delta\Phi) \\ &- 2AB\sin(\delta\omega\tau + \omega\Delta\tau_{RF} + \omega_{IF}\Delta\tau_{IF} + \Delta\Phi)\sin(\phi_A - \phi_B + \omega t_{1/4})], \end{aligned} \quad (C4)$$

where $\delta\omega$ is the baseband frequency of the monochromatic signal and the various phase terms are $\Delta\tau_{RF} \equiv \alpha_L - \alpha_R$, $\Delta\tau_{IF} \equiv \beta_L - \beta_R$, and $\Delta\Phi \equiv \phi_L + \psi_L - \phi_R - \psi_R$. The symmetric part of $L(\tau)$ is $Q(\tau)$ while the antisymmetric part gives $U(\tau)$.

Consider a 100% linearly polarized signal, for which $\phi_A = \phi_B$, and the amplitudes are $A = \cos\chi_\omega$ and $B = \cos\chi_\omega$ where χ_ω is the orientation of the plane of polarization. For this case the Stokes correlations are

$$\begin{aligned} I(\tau) &= \cos(\delta\omega\tau) \\ L(\tau) &= \cos(\delta\omega\tau + \omega\Delta\tau_{RF} + \omega_{IF}\Delta\tau_{IF} + \Delta\Phi + 2\chi_\omega)\,. \\ V(\tau) &= 0 \end{aligned} \quad (C5)$$

Equation (C5) demonstrates that the instrumental time delays result in rotation of the plane of polarization. The measured polarization angle is

$$\tilde{\chi}_\omega = \chi_\omega + (\omega\Delta\tau_{RF} + \omega_{IF}\Delta\tau_{IF} + \Delta\Phi)/2.$$

By measuring the differential rotation of χ_ω across a bandwidth $\Delta\omega$, the total delay may be solved as

$$\Delta\tau \equiv \Delta\tau_{RF} + \Delta\tau_{IF} = 2(\Delta\tilde{\chi}_\omega - \Delta\chi_\omega)/\Delta\omega.$$

The depolarization across a rectangular bandpass is obtained by assuming that Faraday rotation in the wavefield is described as $\chi_\omega = \chi_0 + \zeta(\omega - \omega_0)$ and integrating $L(\tau)$ over frequency (since second moments sum for an incoherent wavefield, which we assume). The result is

$$L_{\Delta\omega}(\tau) = \Delta\omega\cos(\delta\omega\tau + \omega_0\Delta\tau_{RF} + \omega_{IF0}\Delta\tau_{IF} + \Delta\Phi + 2\chi_0)\left(\frac{\sin x}{x}\right), \quad (C6)$$

where $x \equiv \Delta\omega(\tau + \Delta\tau + 2\zeta)/2$. Equation (C6) implies that for observations over a single bandwidth one must have $x \leq 0.55$ in order for the depolarization factor to be larger than 95%. Single bandwidth observations (such as those using an adding polarimeter) correspond to setting $\tau = 0$ in equation (C6), because only a single lag of cross correlation is provided. For correlator polarimetry, the depolarization factor may be evaluated letting $\Delta\omega$ equal the width of each frequency channel.

The Fourier transforms of the correlation functions yield the Stokes parameters for each frequency channel. Figure 8 shows the resultant position angle plotted against frequency channel when a time delay of 40 ns was introduced (unintentionally) between the right and left hand signal paths. The rotation with frequency channel results in nearly complete depolarization of the linear polarization if Q and U are not first derotated.