The Measurement Process with Fully-Filled Apertures

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Accuracy in Radio Astronomy

- Pulsar Periods: 1 part in $10^{12}$
- HI Redshifts: For $z \approx 0.1$, about 1 part in $10^4$
- Source Intensities and Polarization Parameters: Typically $\approx \pm 10\%$

Basic Definitions

A: Intensity/Surface Brightness:

$$I(\theta, \phi, \nu, t) = B(\theta, \phi, \nu, t) = \frac{1}{4\pi} \int \frac{dE}{dA \, dA \, d\Omega}$$

Standard units are W m$^{-2}$ Hz$^{-1}$ ster$^{-1}$.

B: Brightness Temperature:

$$T_B = \frac{2k\lambda^2}{c^2} \int \frac{dE}{dA \, dA \, d\Omega}$$

where $\lambda$ is the wavelength, $k$ is Boltzmann’s constant, and $c$ is the velocity of light.

C: Flux Density

Now, the Flux Density $S$ is given by:

$$S(\theta, \phi, \nu, t) = \frac{1}{4\pi} \int \frac{dE}{dA \, dA \, d\Omega}$$

Hence, if we consider our tiny piece of sky as a discrete source,

$$S(\theta, \phi, \nu, t) = \frac{dE}{dA \, dA \, d\Omega} = \frac{dE}{dA \, dA \, d\Omega} = \frac{dE}{dA \, dA \, d\Omega}$$

The standard units of flux density are W m$^{-2}$ Hz$^{-1}$.

However, a practical unit, the JANSKY, has been adopted, where $1 \text{ Jy} = 10^{-26}$ W m$^{-2}$ Hz$^{-1}$.

With $1 \text{ mJy} = 10^{-5} \text{ Jy}$ and $1 \text{ Jy} = 10^{-5} \text{ Jy}$.

D: Distance Dependencies

If our small radio source is situated at a distance $r$, then by the inverse square law, the energy we receive from it is $dE \propto r^{-2}$. However, the solid angle that it subtends is $d\Omega \propto r^2$. Hence, $I = dE/(d\Omega \, dA \, d\Omega)$ is distance independent.

In contrast, the flux density $S = dE/(d\Omega \, dA \, d\Omega)$ is $r^{-2}$, and hence the flux density of a “standard candle” falls as the inverse square of its distance.
The Basic Radio Receiver

Celestial radio emission can be subdivided into two main spectral categories:

1. Broadband or Continuum Emission.
   Typical examples: (a) planetary surfaces, (b) HII regions, (c) the Cosmic Microwave Background & (d) those Galactic and extragalactic sources that emit via the synchrotron process.

2. Line emission due to low-energy transitions within atoms and molecules in space.

The Super-Heterodyne Receiver

Mixer: Why frequency change rather than amplify at \( \nu_{RF} \) all the way down the receiver chain?

1. Long cables are less lossy at lower frequencies. (Nowadays less important due to use of optical fibers.
2. Fear of positive feedback causing receiver oscillations.
3. If \( \nu_{LO} \) can be varied over a wide range, a standard, fixed frequency, IF chain can service a wide range of receiver front-ends.

Detector:
The detector brings the band-center frequency down to DC (0 Hz). Usually a square-law detector is employed, for which:

\[
\text{Output Voltage} \propto (\text{Input Voltage})^2
\]
\[
\propto \text{Input Power}
\]

Signal-to-Noise Considerations

System Noise:

Cascaded Amplifiers:

If our receiver chain consists of \( N \) amplifiers, each with gain \( g_i \) and noise temperature \( T_i \), the total system gain \( g \) is

\[
g = g_1 \cdot g_2 \cdots g_N = g_1 \cdot (g_2 \cdot g_3 \cdots g_N)
\]

and the effective receiver temperature is

\[
T' = T_1 + T_2 + T_3 + \cdots + T_N
\]

(As each \( g_i \gg 1 \), clearly \( T' \approx T_1 \))
Lossy Components

The System Temperature

There are extra noise-power contributions from such things as:

- Emission from the neutral atmosphere.
- Ground radiation in the far sidelobes.
- Celestial noise contributions from:
  - the background radiation of the Milky Way.
  - the 2.7 K radiation of the Cosmic Microwave Background.
  - possibly a contribution from the target source itself!

The sum of all these noise contributions is the SYSTEM TEMPERATURE, $T_{sys}$, expressed as:

$$T_{sys} = T_R + T_{esd} + T_{e0} + T_{e1} + T_{e2} + T_{e3}$$

Where the sum of all contributions from outside the observing system is called the ANTENNA TEMPERATURE, $T_A$.

The Radiometer Equation

How weak a source can we detect with a given receiver? For our total-power receiver, the RADIOMETER EQUATION gives the rms noise ripple against which we try to detect our target source, in units of temperature:

$$T_{sys} = \frac{T_{sys}}{\Delta T_A}$$

where $\Delta T_A$ is the bandwidth in Hz, and $\tau$ is the integration time in sec.

Moving our telescope beam ON and OFF a source which gives an antenna temperature rise of $\Delta T_A$, with a dwell time $\tau/2$ in each phase, signal-to-noise ratio will be:

$$\Delta T_A = \frac{T_{sys}}{2\tau T_{sys}}$$

Usually a signal-to-noise ratio of $\sim 6$ is considered necessary for a significant detection.

When considering a point-source flux density, B rather than its $T_A$, we know that the power received on a collecting area $A_{col}$ from the source in a single polarization is $1/2 B A_{col}.$ The same power would be received were the feed to be replaced by a matched resistor of temperature $\Delta T_A.$

$\Delta T_A \propto B$
Single-Dish Imaging and Sampling

How far apart should we place our scans (and how often should we sample along a scan) in order to lose no information which our telescope is capable of passing?

Let us consider a one-dimensional case:

If the normalized beam pattern of the telescope is $B(\theta)$, and the true distribution of brightness temperature is $T_b(\theta)$, the antenna temperature recorded is:

$$T_a(\theta) = \frac{1}{2\pi} \int T_b(\theta') B(\theta - \theta') d\theta'$$

This is a convolution, and the Fourier Transform of a convolution is equal to the multiplication of the Fourier Transforms of the two functions being convolved.

Convolution:

Sampling at discrete points is multiplying the recorded $T_a(\theta)$'s by a set of delta functions spaced by the sampling interval, $\Delta \theta$. This is equivalent to convolving the Fourier transforms of $T_a(\theta)$ (zero outside ±D/3 cycles/deg), by a series of delta functions spaced by $\Delta \theta$ cycles/deg.

The series of “islands” do not overlap, i.e., we retain full information, only if:

$$\frac{\lambda}{2\Delta \theta} \leq \frac{2D}{\lambda}$$

Or in other words:

$$\Delta \theta \leq \frac{\lambda}{2D}$$

Sky Plane

Spatial Sampling Plane

Δθ - Telescope Diameter
L - Wavelength