The NAIC-NRAO School on Single-Dish Radio Astronomy
Techniques and Applications -- July 2005

The Measurement Process with Fully-Filled Apertures

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Accuracy in Radio Astronomy

- Pulsar Periods: 1 part in 10^{12}
- HI Redshifts: For $z \approx 0.1$, about 1 part in 10^4
- Source Intensities and Polarization Parameters: Typically $\approx \pm 10\%$

Basic Definitions

A: Intensity/Surface Brightness:

$I(\theta, \phi, \nu, t)$, where,

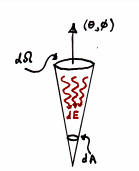
θ and ϕ are the spherical coordinates of a given direction on the two-dimensional sphere.

ν is the frequency.

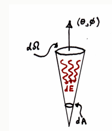
and, t is time, i.e. the emission may be time variable.

$$I(\theta, \phi, \nu, t) = B(\theta, \phi, \nu, t) = \lim_{dt, dA, d\nu, d\Omega \to 0} \frac{dE(\theta, \phi, \nu, t)}{dt\ dA\ d\nu\ d\Omega}$$

Standard units are $W\,m^{-2}Hz^{-1}ster^{-1}$.



B: Brightness Temperature:



T_B is a function of frequency, and related to the Intensity, $I(\theta, \phi, \nu, t)$, by the Rayleigh-Jeans approximation (unless the frequency is very high, and the temperature very low).

$$I(\theta, \phi, \nu, t) = \frac{2\,k\,T_B(\theta, \phi, \nu, t)}{\lambda^2} = \frac{2\,k\,\nu^2\,T_B(\theta, \phi, \nu, t)}{c^2}$$

where, λ is the wavelength, k is Boltzmann's constant, and c is the velocity of light.

C: Flux Density

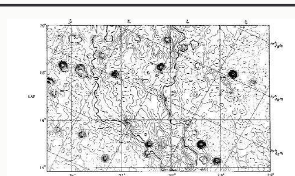


Figure 1: Contour maps of the intermediate latitude total-intensity emission from the North Polar Spur at 1.4 GHz made with the Effelsberg telescope (Softe & Reich, 1979, A&AS, 38, 251). The image is in Galactic coordinates, and the HPBW~ 10 arcmin.

Now, the Flux Density, S, is given by,

$$S(\nu, t) = \iint_{source} I(\theta, \phi, \nu, t)\ d\Omega$$

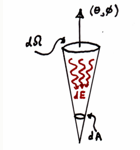
Hence, if we consider our tiny cone of sky as a discrete source,

$$S(\nu, t) = I(\theta, \phi, \nu, t)\ d\Omega = \frac{dE(\theta, \phi, \nu, t)}{dt\ dA\ d\nu\ d\Omega} \times d\Omega = \lim_{dt, dA, d\nu \to 0} \frac{dE(\theta, \phi, \nu, t)}{dt\ dA\ d\nu}$$

The standard units of flux density are $W\,m^{-2}\,Hz^{-1}$.

However, a practical unit, the JANSKY, has been adopted, where, $1\,Jy = 10^{-26}\,W\,m^{-2}Hz^{-1}$.

With, $1\,mJy = 10^{-3}\,Jy$ and $1\,\mu Jy = 10^{-6}\,Jy$.



D: Distance Dependencies

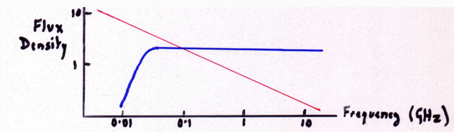
If our small radio source is situated at a distance r, then by the inverse square law, the energy we receive from it is $dE \propto r^{-2}$. However, the solid angle that it subtends is $d\Omega \propto r^{-2}$. Hence, $I = dE/(dt\,dA\,d\nu\,d\Omega)$ is distance independent.

In contrast, the flux density, $S = dE/(dt\,dA\,d\nu) \propto r^{-2}$, and hence the flux density of a "standard candle" falls as the inverse square of its distance.

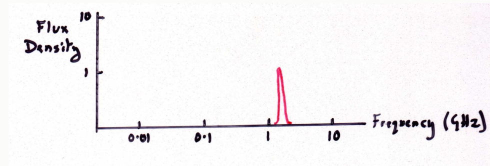
The Basic Radio Receiver

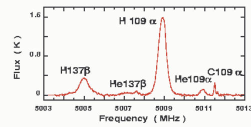
Celestial radio emission can be subdivided into two main spectral categories:

1. Broadband or Continuum Emission.
   Typical examples: (a) planetary surfaces, (b) HII regions, (c) the Cosmic Microwave Background & (d) those Galactic and extragalactic sources that emit via the synchrotron process.



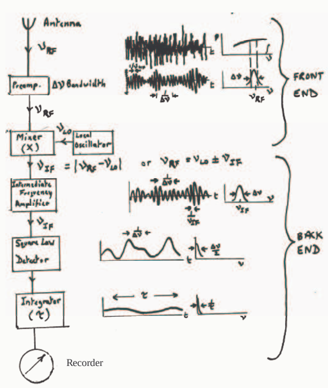
2. Line emission due to low-energy transitions within atoms and molecules in space.





Terzian & Lewis, (unpublished)

The Super-Heterodyne Receiver



Mixer: Why frequency change rather than amplify at ν_{RF} all the way down the receiver chain?

1. Long cables are less lossy at lower frequencies. (Nowadays less important due to use of optical fibers.
2. Fear of positive feedback causing receiver oscillations.
3. If ν_{LO} can be varied over a wide range, a standard, fixed frequency, IF chain can service a wide range of receiver front-ends.

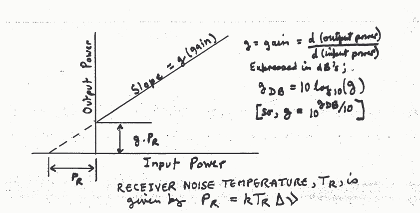
Detector:

The detector brings the band-center frequency down to DC (0 Hz). Usually. a square-law detector is employed. for which.

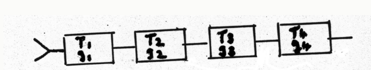
$$\text{Output Voltage} \propto (\text{InputVoltage})^2$$
$$\propto \text{InputPower}$$

Signal-to-Noise Considerations

System Noise:



Cascaded Amplifiers:



If our receiver chain consists of N amplifiers. each with gain. g_i. and noise temperature. T_i. the total system gain. g. is.

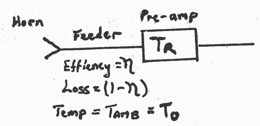
$$g = g_1 \times g_2 \times g_3 \times \ldots \times g_N$$

and the effective receiver temperature is.

$$T_R = T_1 + \frac{T_2}{g_1} + \frac{T_3}{g_1 g_2} + \frac{T_4}{g_1 g_2 g_3} + \ldots$$

(As each $g_i \gg 1$. clearly. $T_R \approx T_1$)

Lossy Components



$$T_R' = (\frac{1}{\eta} - 1)\, T_o + \frac{T_R}{\eta}$$

(As $\eta < 1$. $T_R' > T_R$)

The System Temperature

There are extra noise-power contributions from such things as;

a) Emission from the neutral atmosphere.
b) Ground radiation in the far sidelobes.
c) Celestial noise contributions from;
 i) the background radiation of the Milky Way.
 ii) the 2.7 K radiation of the Cosmic Microwave Background.
 iii) possibly a contribution from the target source itself!

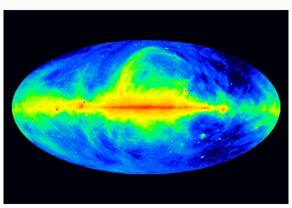
The sum of all these noise contributions is the SYSTEM TEMPERATURE. T_{sys}. expressed as.

$$T_{sys} = T_R + \underbrace{T_{atm} + T_{ground} + T_{gal} + T_{CMB} + T_{source}}_{T_A}$$

Where the sum of all contributions from outside the observing system is called the ANTENNA TEMPERATURE, T_A.

The contribution from the Milky Way galaxy plus the CMB changes greatly with frequency. i.e.

- at 38 MHz. $T_{gal} \geq 10^4 - 10^5$ K
- at 327 MHz. $T_{gal} \sim 20 - 2{,}000$ K
- at 5 GHz and above. T_{gal} is negligible. and is dwarfed by $T_{CMB} = 2.7$ K



The Radiometer Equation

How weak a source can we detect with a given receiver? For our total-power receiver, the RADIOMETER EQUATION gives the rms noise ripple against which we try to detect our target source, In units of temperature;

$$T_{rms} = \frac{T_{sys}}{\sqrt{\Delta\nu\ \tau}}$$

where. $\Delta\nu$ is the bandwidth in Hz. and τ is the integration time in sec.

Moving our telescope beam ON and OFF a source which gives an antenna temperature rise of ΔT_A. with a dwell time $\tau/2$ in each phase. signal-to-noise ratio will be.

$$\frac{\Delta T_s}{T_{rms}} = \frac{\Delta T_A \sqrt{\Delta\nu\ \tau}}{2 \times T_{sys}}$$

Usually a signal-to-noise ratio of ~5 is considered necessary for a significant detection.

When considering a point source's flux density. S. rather than its T_A. we know that the power received on a collecting area. A_{eff}. from the source in a single polarization is $1/2\, S\, A_{eff}\, \Delta\nu$. The same power would be received were the feed to be replaced by a matched resistor of temperature. ΔT_A. so.

$$\frac{1}{2} S\, A_{eff} \Delta\nu = k \Delta T_A \Delta\nu$$

or.

$$\Delta T_A = \frac{S A_{eff}}{2k}$$

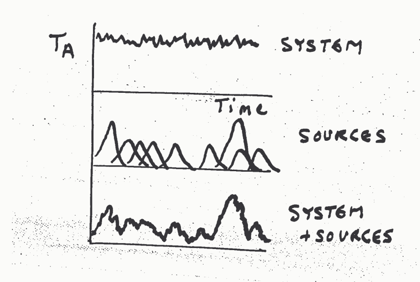
It often simpler when considering sensitivity in terms of point-source flux density to express T_{sys} in terms of the flux density that would exactly double the system temperature. This is called the SYSTEM EQUIVALENT FLUX DENSITY (SEFD). and is given by

$$SEFD = \frac{2\, k\, T_{sys}}{A_{eff}}$$

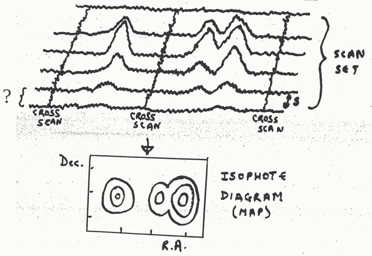
Then. the radiometer equation becomes.

$$S_{rms} = \frac{SEFD}{\sqrt{\Delta\nu\ \tau}}$$

CONFUSION



Single-Dish Imaging and Sampling



How far apart should we place our scans (and how often should We sample along a scan) in order to lose no information which our telescope is capable of passing?

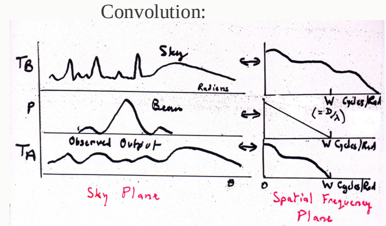
Let us consider a one-dimensional case;

If the normalized beam pattern of the telescope is $P(\theta)$, and the true distribution of brightness temperature is $T_B(\theta)$, the antenna temperature recorded is,

$$T_A(\theta') = \frac{1}{\Omega_A}\int T_B(\theta)\, P(\theta' - \theta)\, d\theta$$

This is a convolution, and the Fourier Transform of a convolution is equal to the multiplication of the Fourier Transforms of the two functions being convolved.

Convolution:



D = Telescope Diameter

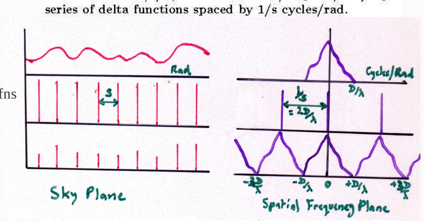
λ = Wavelength

Sampling at discrete points is multiplying the recorded $T_A(\theta')$ by a set of delta functions separated by the sampling interval, s rads. This is equivalent to convolving the Fourier transform of $T_A(\theta')$, (zero outside $\pm$ D/λ cycles/rad), by a series of delta functions spaced by 1/s cycles/rad.

T_A

Delta-fns

X



The series of "islands" do not overlap, i.e. we retain full information, only if,

$$\frac{1}{s} > \frac{2D}{\lambda}$$

Or in other words,

$$s < \frac{\lambda}{2D}\ \mathrm{rad}$$