SPECTRAL BASELINES

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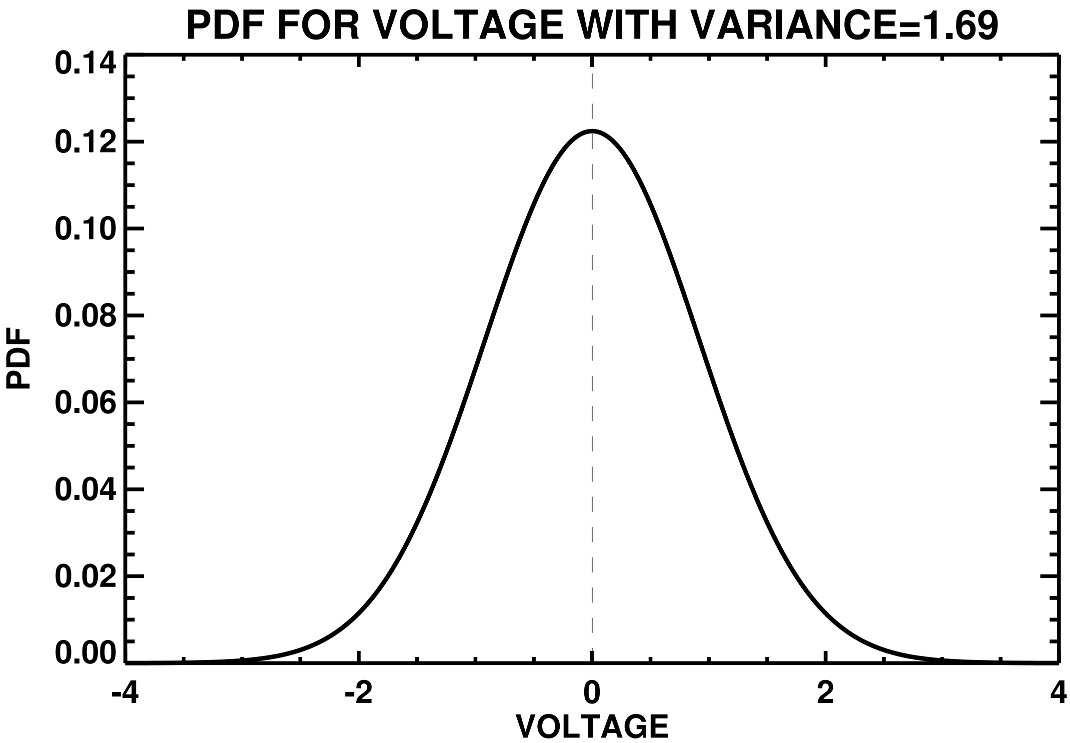
VOLTAGE AND POWER

Zillions of uncorrelated wiggling electrons produce a random wiggly electric field. The *Central Limit Theorem* **says that the resultant field has Gaussian statistics. Gaussian statistics contaain two, and only two, parameters:**

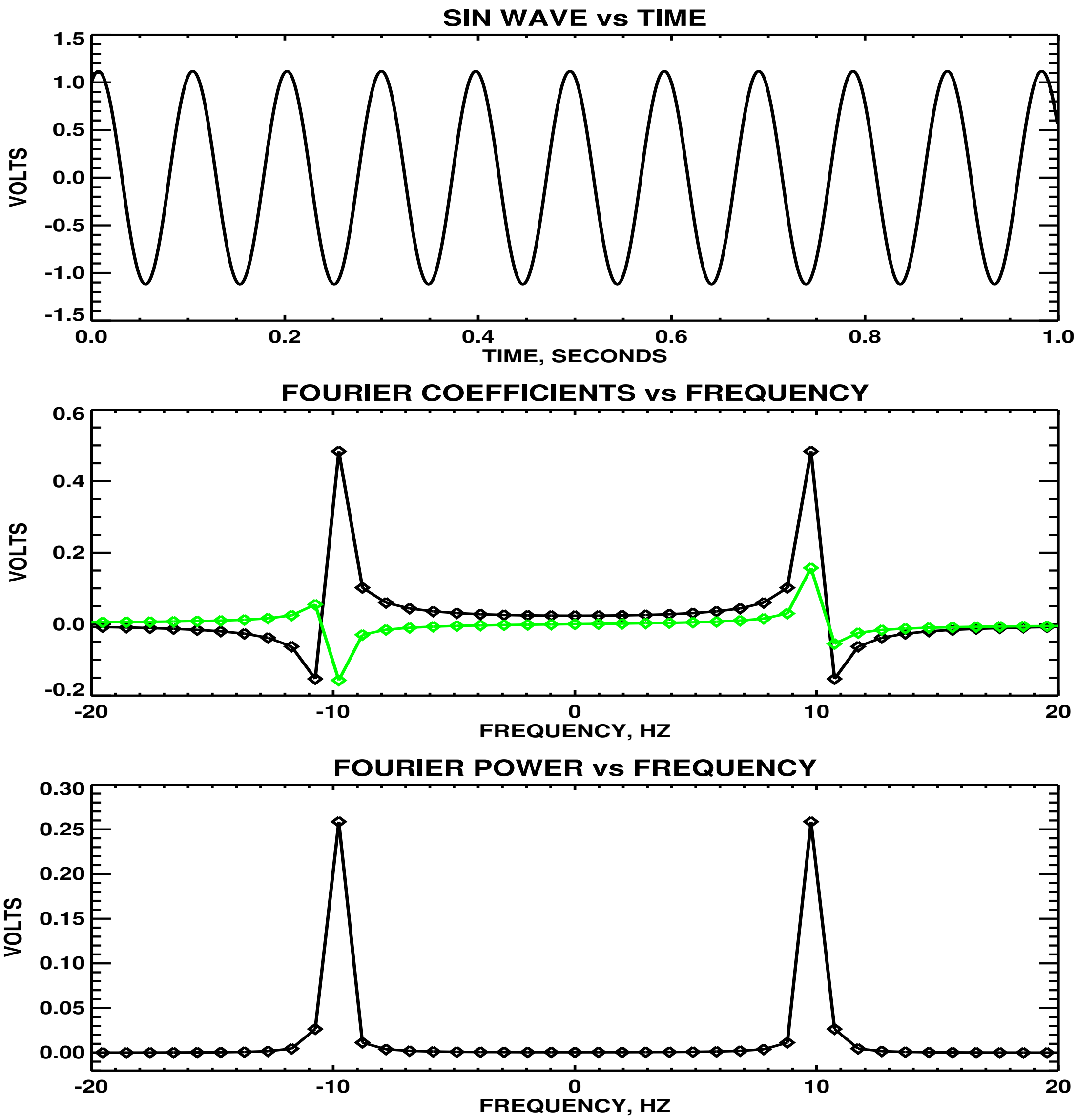
- **The** *mean***, which is zero (a DC electric field doesn't propagate).**
- **The** *variance***, which measures the power density, equal to** $\langle V^2 \rangle$**.**



To measure V^2 we must square the voltage and measure its time average. This process is called *detection*.

HOW ABOUT THE SPECTRUM?

A spectral feature has excess power at some frequencies. The spectral shape is revealed by the *Fourier transform***. The example below shows a monochromatic signal and its Fourier transform. You have to square the Fourier coefficients to get the power spectrum!**


SIN WAVE vs TIME
VOLTS
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
0.0
0.2
0.4
0.6
0.8
1.0
TIME, SECONDS
FOURIER COEFFICIENTS vs FREQUENCY
VOLTS
0.6
0.4
0.2
0.0
-0.2
-20
-10
0
10
20
FREQUENCY, HZ
FOURIER POWER vs FREQUENCY
VOLTS
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-20
-10
0
10
20
FREQUENCY, HZ

THE CORRELATION THEOREM

Above we said (graphically). . .

THE POWER SPECTRUM IS THE QUADRATIC SUM OF THE REAL/IMAG FOURIER COEFFICIENTS

In Fourier transforms, the correlation theorem says. . .

THE POWER SPECTRUM IS THE FOURIER TRANSFORM OF THE AUTOCORRELATION FUNCTION

The autocorrelation function $A(\tau)$ is

$$A(\tau) = \int V(t) \cdot V(t + \tau) dt$$

So you can calculate the power spectrum in *two ways!*

- **FX: Fourier transform, then square. The Fast Fourier transform (FFT) does this in $(N \ln_2 N)$ arithmetic operations.**

- **XF: Square (autocorrelate), then Fourier transform. This takes (N^2) operations.**

For large N, FX has many fewer ops. But *historically*, correlators are used: it's easy to parallelize the autocorrelation calculation in hardware.

We are on the cusp of new technology: now, FPGA chips make FX cheaper and faster. The GALFA spectrometer and upcoming Arecibo spectrometers are FX.

TWO IMPORTANT POINTS ABOUT DIGITAL SPECTROMETERS

Digital spectrometers are wonderful but have two important requirments:

- **The maximum input frequency must be less than half the sample frequency. This is based on the** *sampling theorem* **and is called the** *Nyquist criterion*. **Violating this leads to** *aliasing*, **which ruins your results. A common example of aliasing is wheels turning backwards in movies.**

  The only way to ensure this is to limit the bandwidth of the input signl with a filter. The gain of this filter varies across the spectrum and must be removed.

- **Digital spectrometers operate at baseband, i.e. at frequencies centered near or around zero (in other words, DC). This is like the human ear, which also works at baseband.**

HETEROGYNE SPECTROSCOPY: THE BASIS OF RADIOS, TV'S, CELL PHONES, AND. . . RADIO ASTRONOMY

This baseband requirement is interesting. For the human ear, how does sound carried by radio waves—say, at 91.7 MHz in the FM band—get converted to baseband? And how does our HI line measurment get converted to baseband?

$$\underbrace{\cos(F_{RF}) \cdot \cos(F_{LO})}_{product} = \frac{1}{2} \cos \underbrace{(F_{RF} - F_{LO})}_{difference} + \frac{1}{2} \cos \underbrace{(F_{RF} + F_{LO})}_{sum}$$

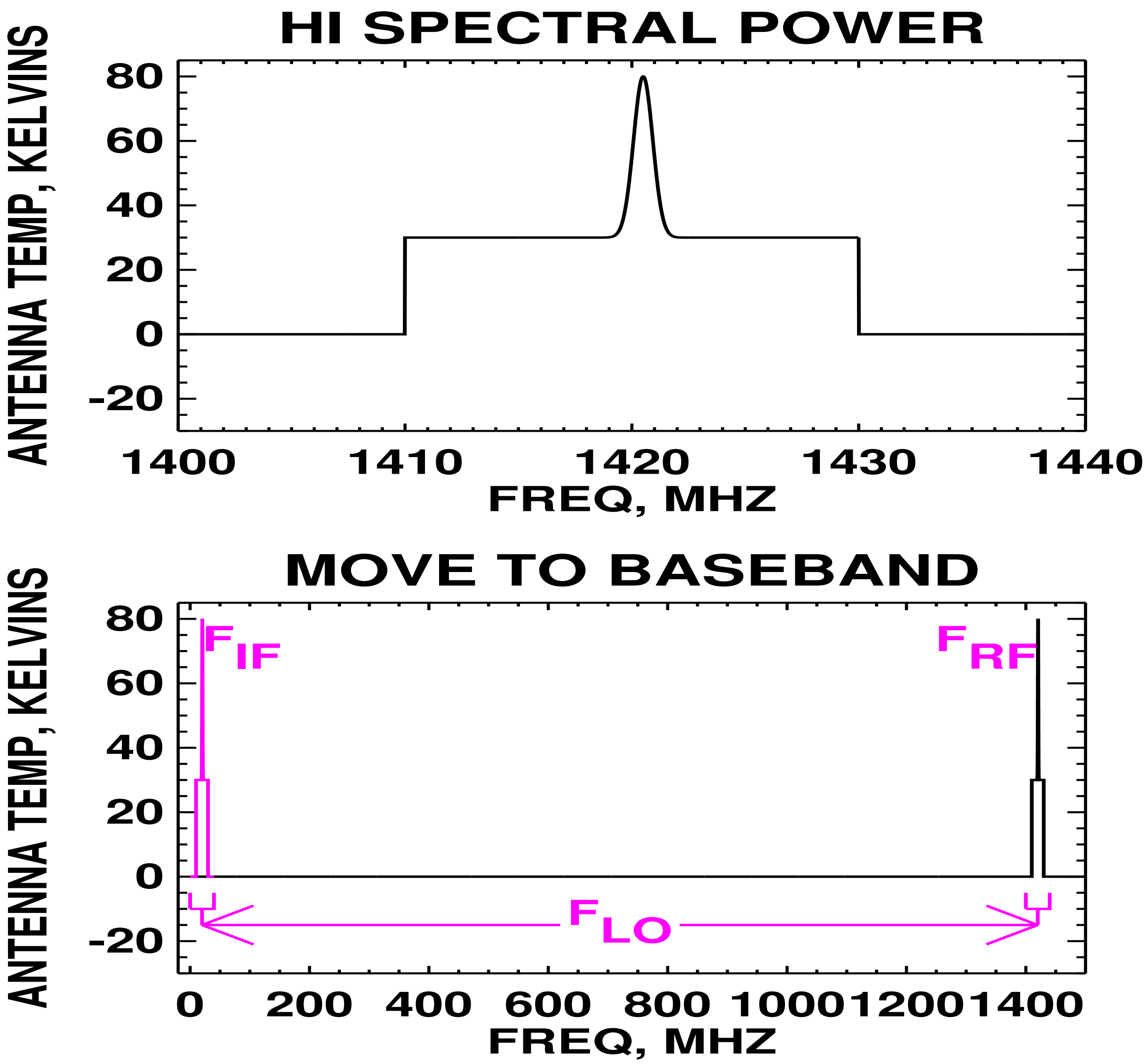
Discard the sum with a filter; then

$$F_{IF} = R_{RF} - F_{LO}$$

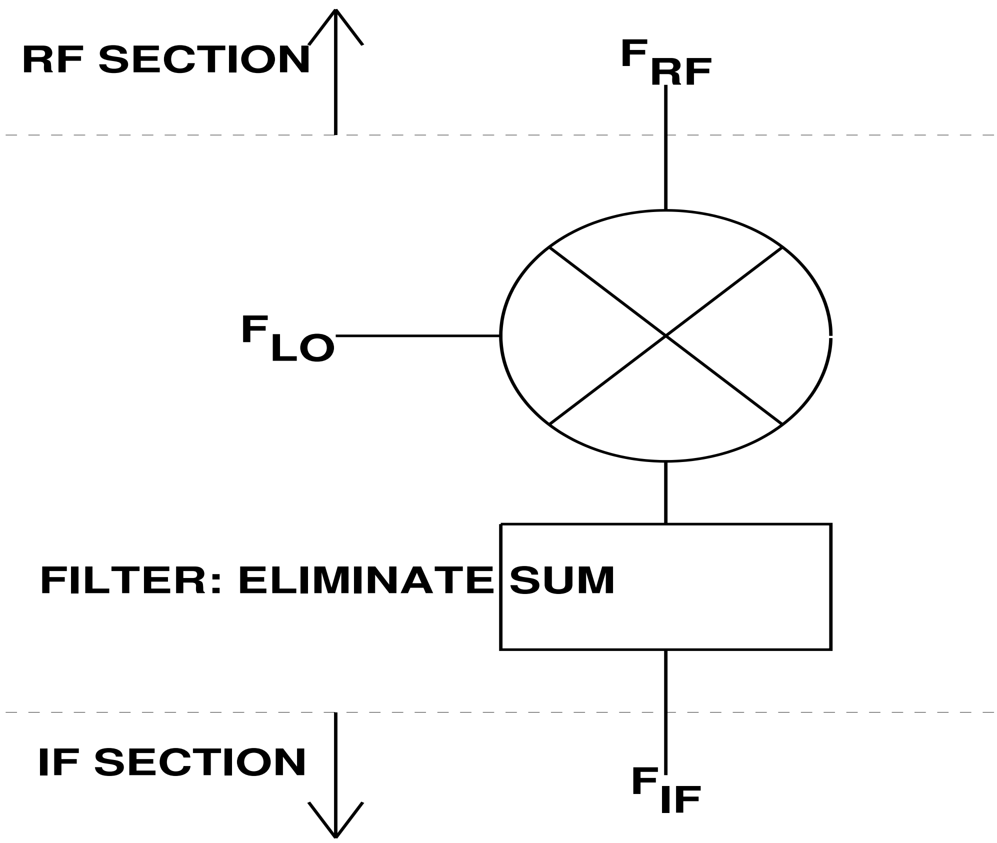
.

Graphically . . .

THE BASIS OF HETERODYNE SPECTROSCOPY



This multiplication process is called *mixing***. The multiplier is called a** *mixer***. The original frequency is called the** *RF frequency* F_{RF}**. The multiplying signal is generated by a local oscillator and called, naturally enough, the** *local oscillator frequency* F_{LO}**. The difference frequency is called the** *intermediate frequency* F_{IF}**. The block diagram:**

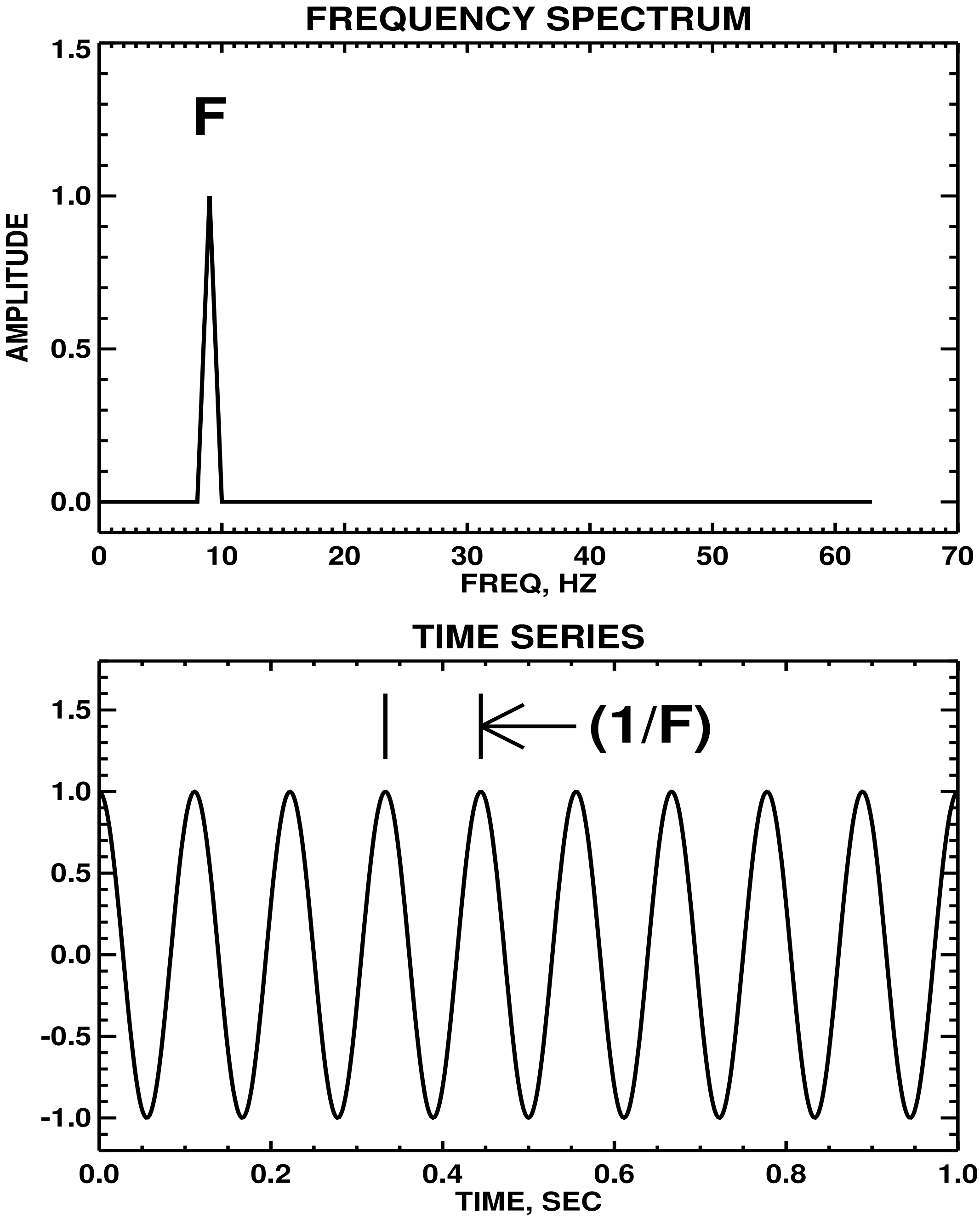

RF SECTION
F RF
F LO
FILTER: ELIMINATE SUM
IF SECTION
F IF

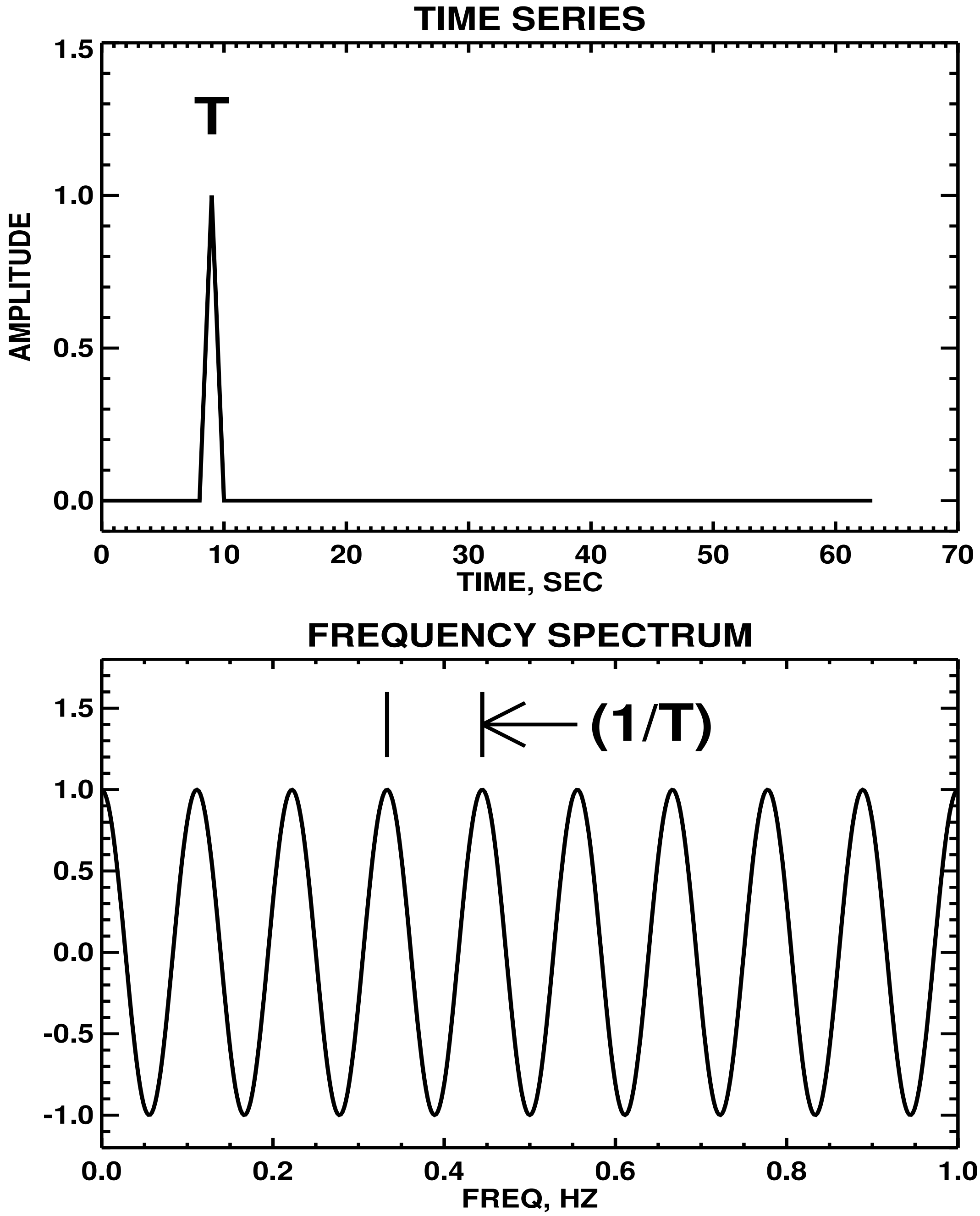
CONTRIBUTIONS FROM THE RF SECTION

Contributions to the observed spectrum from the RF section include the object of interest and everything else. The other stuff includes both added noise spectrally-dependent gain. These, in turn, are either benign or malignant instrumental effects. The benign ones are easily dealt with; the malignant ones are not. They include:

- **Good Astronomy! Continuum and spectral radiation from the sky, in the direction where the telescope is pointed.**
- **Bad Astronomy! Spectral radiation from the sky, *not* in the direction where the telescope is pointed. Gives false spectral features. In HI called *stray radiation*.**
- **Bad Astronomy! Continuum radiation from the sky reflected from structural components into the feed. At Arecibo, during daytime the Sun produces the famous *Arecibo Ripple*.**

- **Terrestrial Interference.**
- **Ground emission making its way into the feed via reflections.**
- **Ground emission making its way into the feed directly without reflections.**
- **Spectrally flat noise added by RF electronics.**
- **Spectrally flat noise added by RF electronics.**
- **Noise with spectral features added by RF electronics.**


FREQUENCY SPECTRUM
F
AMPLITUDE
1.5
1.0
0.5
0.0
0
10
20
30
40
50
60
70
FREQ, HZ
TIME SERIES
(1/F)
1.5
1.0
0.5
0.0
-0.5
-1.0
0.0
0.2
0.4
0.6
0.8
1.0
TIME, SEC


TIME SERIES
T
AMPLITUDE
1.5
1.0
0.5
0.0
0
10
20
30
40
50
60
70
TIME, SEC
FREQUENCY SPECTRUM
(1/T)
1.5
1.0
0.5
0.0
-0.5
-1.0
0.0
0.2
0.4
0.6
0.8
1.0
FREQ, HZ

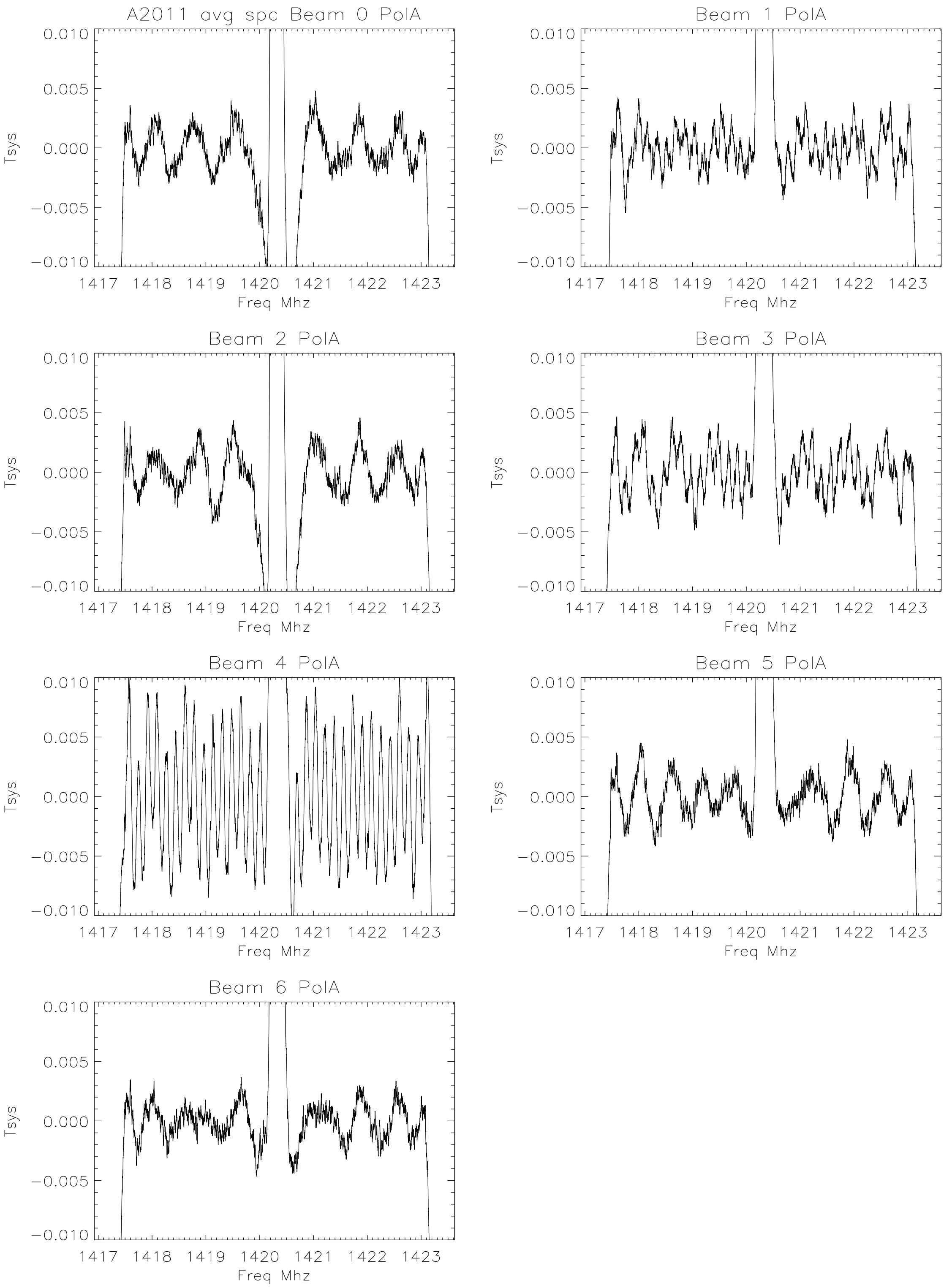
WHY ARE REFLECTIONS SO MALIGNANT?

Suppose a signal enters the feed from two paths whose lengths differ by L. Then the signal adds to a delayed copy of itself; the time delay is $\frac{L}{c}$. This means its autocorrelation function has a sharp spike at time delay $\frac{L}{c}$.

Remember the correlation theorem: The power spectrum is the Fourier transform of the Autocorrelation function. In Fourier space, a spike in one coordinate leads to a sinuosidal ripple in the conjugate coordinate. The spike at delay $\frac{L}{c}$ gives a sinuosidal ripple in the spectrum with period $\frac{c}{L}$.

At Arecibo, the distance between the feed and the bowl is about 130 meters. A signal that reflects off of the feed back down and makes one round trip to return travels 260 meters and suffers a delay of about 0.9 μs. The period of the ripple is about 1.1 MHz. This is equivalent to about 200 km s^{-1} at the HI line.

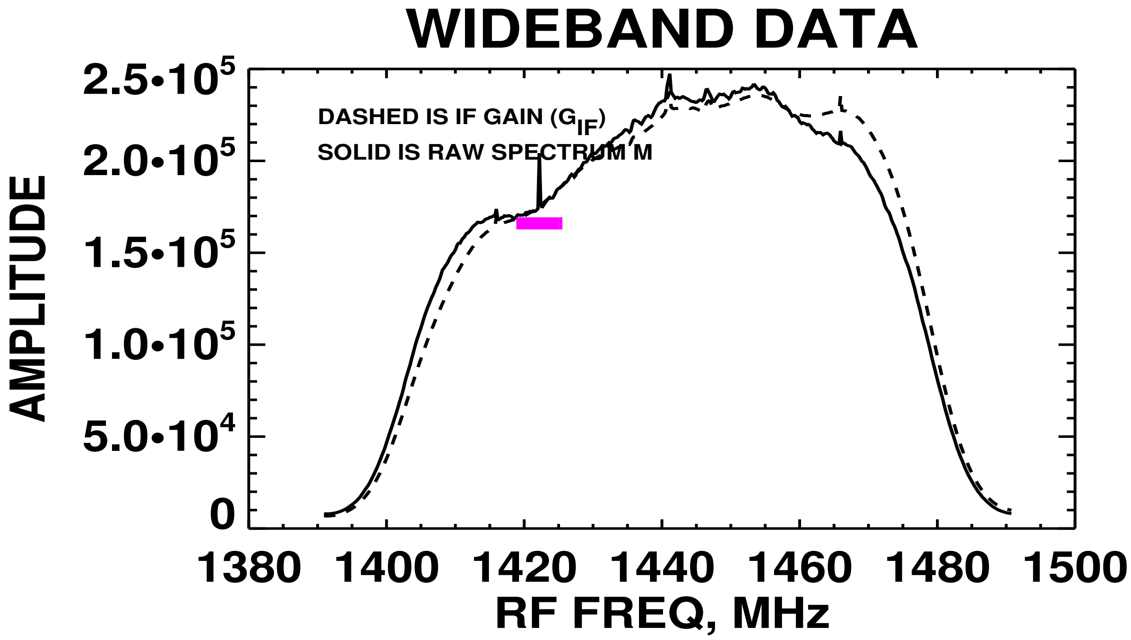
Lots of objects in the sky have lines comparable to this width—the Milky Way galaxy, for example. This is a MAJOR PROBLEM for 21-cm line astronomy. In the one respect it would be nice if the Arecibo telescope were much smaller so the ripple would be much broader(!).

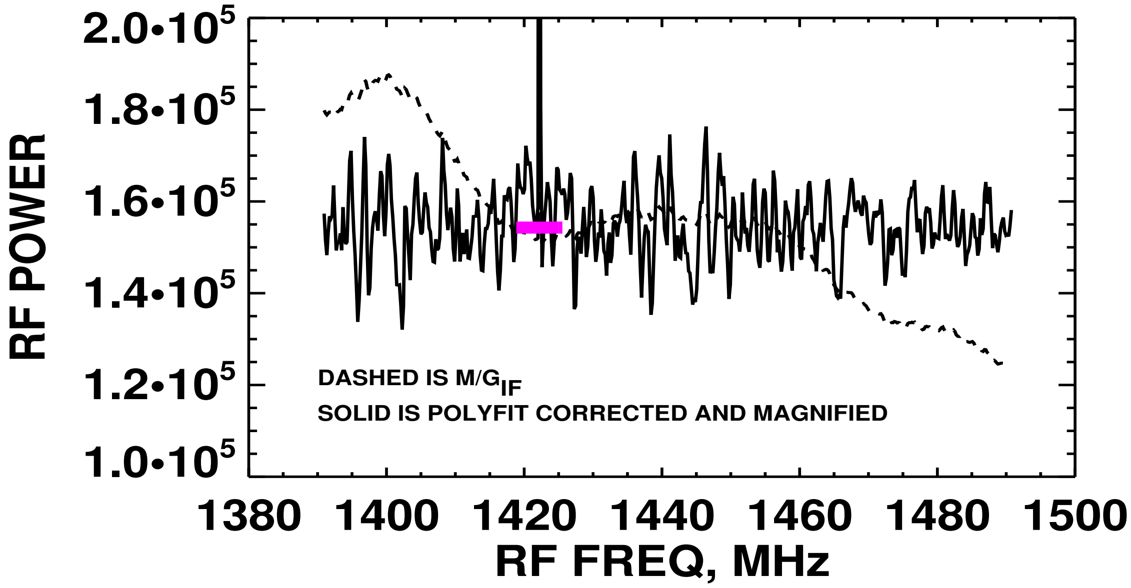

A2011 avg spc Beam 0 PolA
Beam 1 PolA
Beam 2 PolA
Beam 3 PolA
Beam 4 PolA
Beam 5 PolA
Beam 6 PolA
Tsys
Freq Mhz
0.010
0.005
0.000
−0.005
−0.010
1417
1418
1419
1420
1421
1422
1423

CONTRIBUTIONS FROM THE IF SECTION

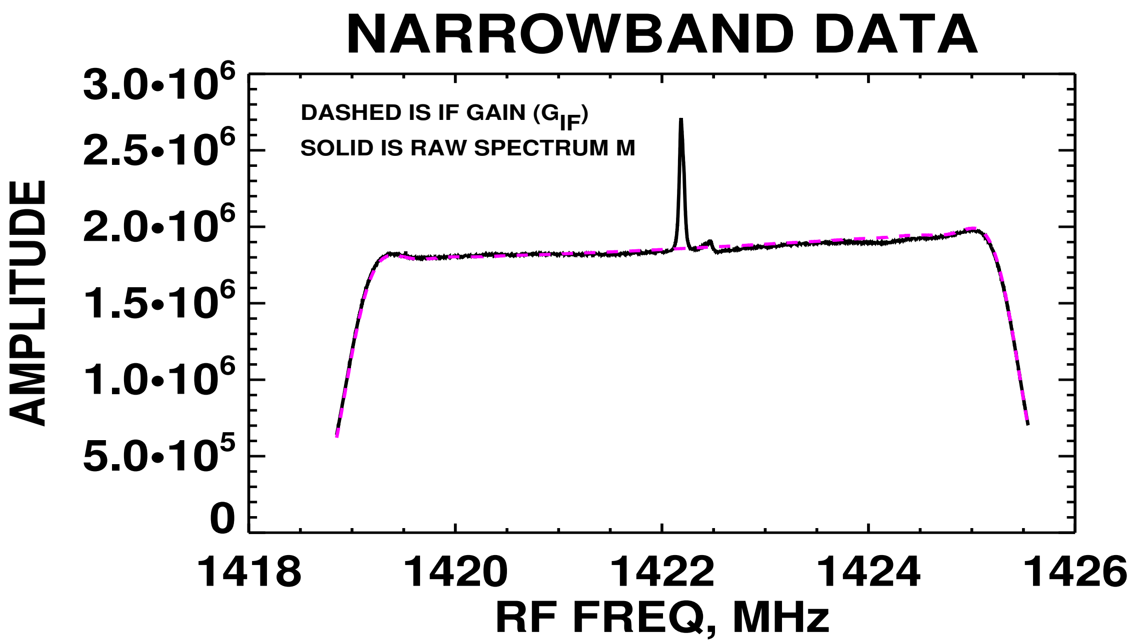
In contrast to the RF section, the IF section adds no noise in a well-designed system. But it has severe spectrally-dependent gain. This is easily dealt with as long at it is time-independent. Unfortunately, this is not always the case: time-dependent effects are always difficult.

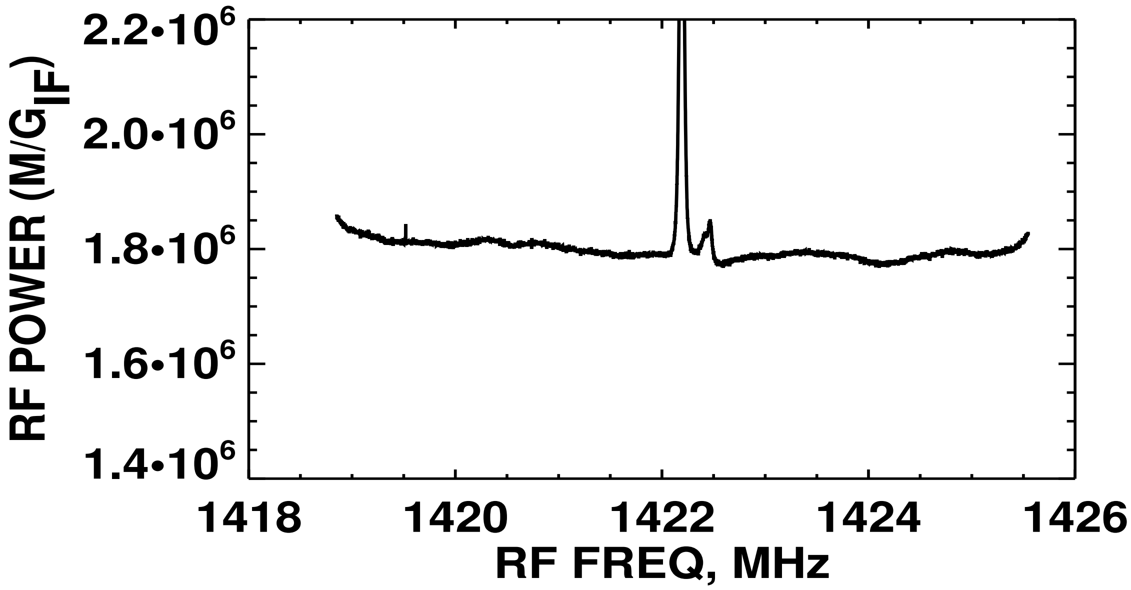
RF POWER AND IF GAIN FOR GALFA WIDEBAND (“CALIBRATION”) SPECTRUM





RF POWER AND IF GAIN FOR GALFA NARROWBAND ("SCIENCE") SPECTRUM





MATHEMATICAL DESCRIPTION

Let:

- $T_*(f_{RF})$ **be the sky spectral line contribution [indicated by** (f)**].**
- T_* **be the sky continuum contribution (no frequency dependence).**
- $T_R(f_{RF})$ **be the front end section spectral contribution.**
- T_R **be the front end section continuum contribution.**
- $G_{RF}(f_{RF})$ **be the RF gain (dependent on RF frequency).**
- $G_{IF}(f_{IF})$ **be the IF gain (dependent on IF frequency).**
- $M(f_{IF})$ **be the measured power spectrum.**
- **unprimed quantities indicate the ON measurement.**
- **primed quantiies indicate the OFF measurement.**

Then we have

$$M(f_{IF}) = G_{IF}(f_{IF})G_{RF}(f_{RF}) \left\{ \underbrace{(T_*(f_{RF}) + T_R(f_{RF}))}_{spectral} + \underbrace{(T_* + T_R)}_{continuum} \right\}$$

Our goal: find $\boldsymbol{T_*(f_{RF})}$.

MATHEMATICAL DESCRIPTION

To this end, we take ON (unprimed) and OFF (primed) spectra. The OFF spectrum evaluates the various contaminating terms. We assume the system contributions are identical for ON and OFF. We assume the system spectral contribution is fractionally small compared to its continuum contribution, i.e. $T_R(f_{RF}) \ll T_R$**, so we can make a Taylor expansion.**

Position switching.

The best OFF is off in *position*. **We combine the ON and OFF as follows. We assume the OFF position has no line, i.e.** $T'_*(f_{RF}) = 0$**)**

$$\frac{M(f_{IF}) - M'(f_{IF})}{M'(f_{IF})} = [T_*(f_{RF}) + (T_* - T'_*)] \left[\frac{1 - \frac{T_R(f_{RF})}{(T'_* + T_R)}}{T'_* + T_R} \right]$$

This gives the ON-OFF spectrum, normalized by the right-hand factor. The frequency-dependence of this factor is fractionally small, so it changes the line shape only a bit and is not serious.

Contaminating influence: **The possibility for** *real problems* **arises if there is a large continuum diffrence between ON and OFF, i.e. if** $(T_* - T'_*)$ **is large. This affects weak line measurements on continuum sources.**

Frequency switching.

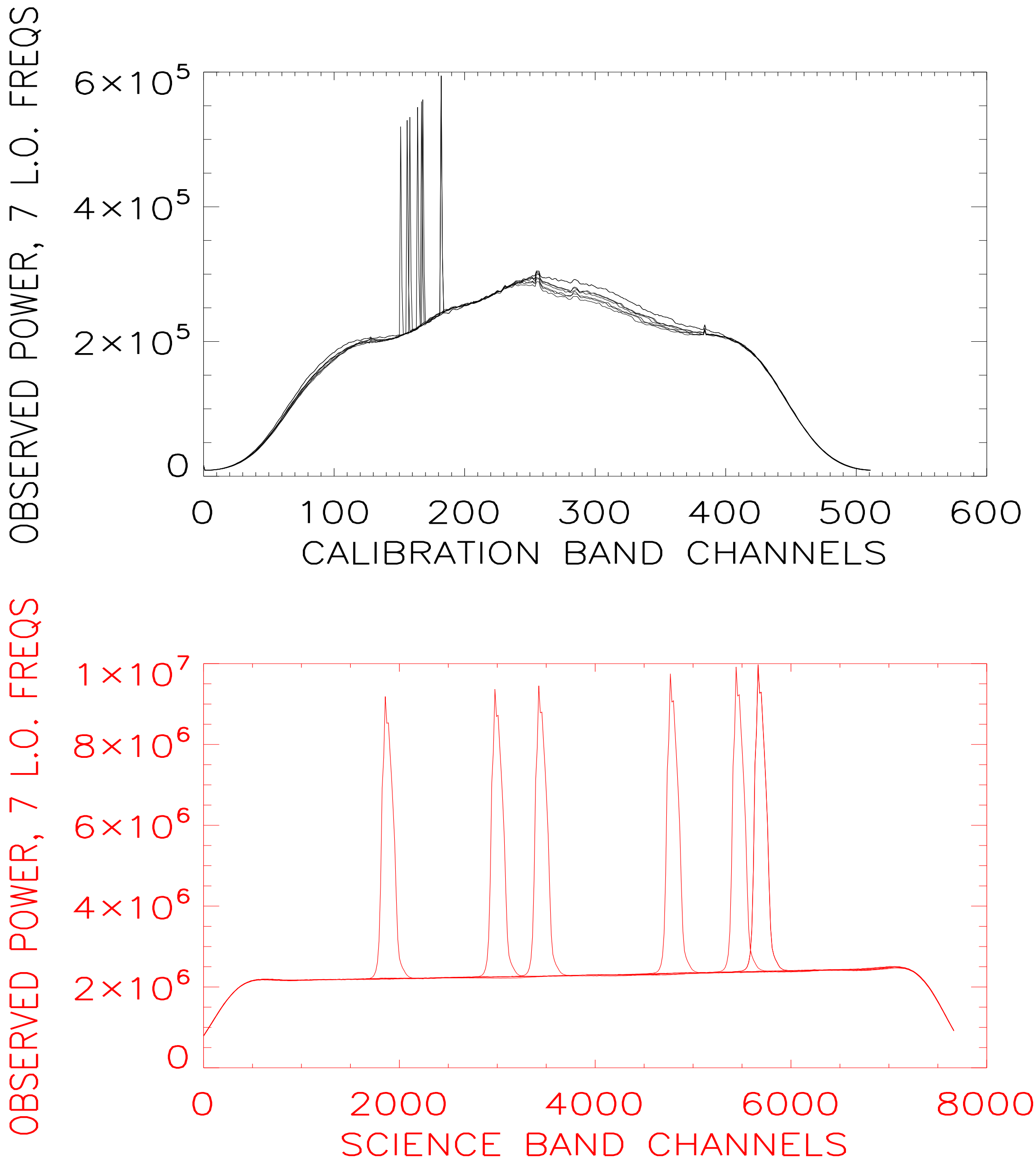
If the spectral line is spatially extended you can't move off in position. Galactic HI is the prime example! So here your OFF is taken off frequency, which means the OFF and ON $G_{RF}(fRF)$ **aren't identical. Retaining only first-order terms, we obtain the previous result plus a complicating term: first factor is replaced by**

$$\left[T_*(f_{RF}) + (T_* - T'_*) + \frac{\delta G}{G}(T_R + 2T_*)\right]$$

where $\frac{\delta G}{G} = 1 - \frac{G'_{RF}(f_{RF})}{G_{RF}(f_{RF})}$**. Even though** $\frac{\delta G}{G} \ll 1$, **it operates on the** *receiver temperature*, **which is large, and produces serious baseline contamination.**

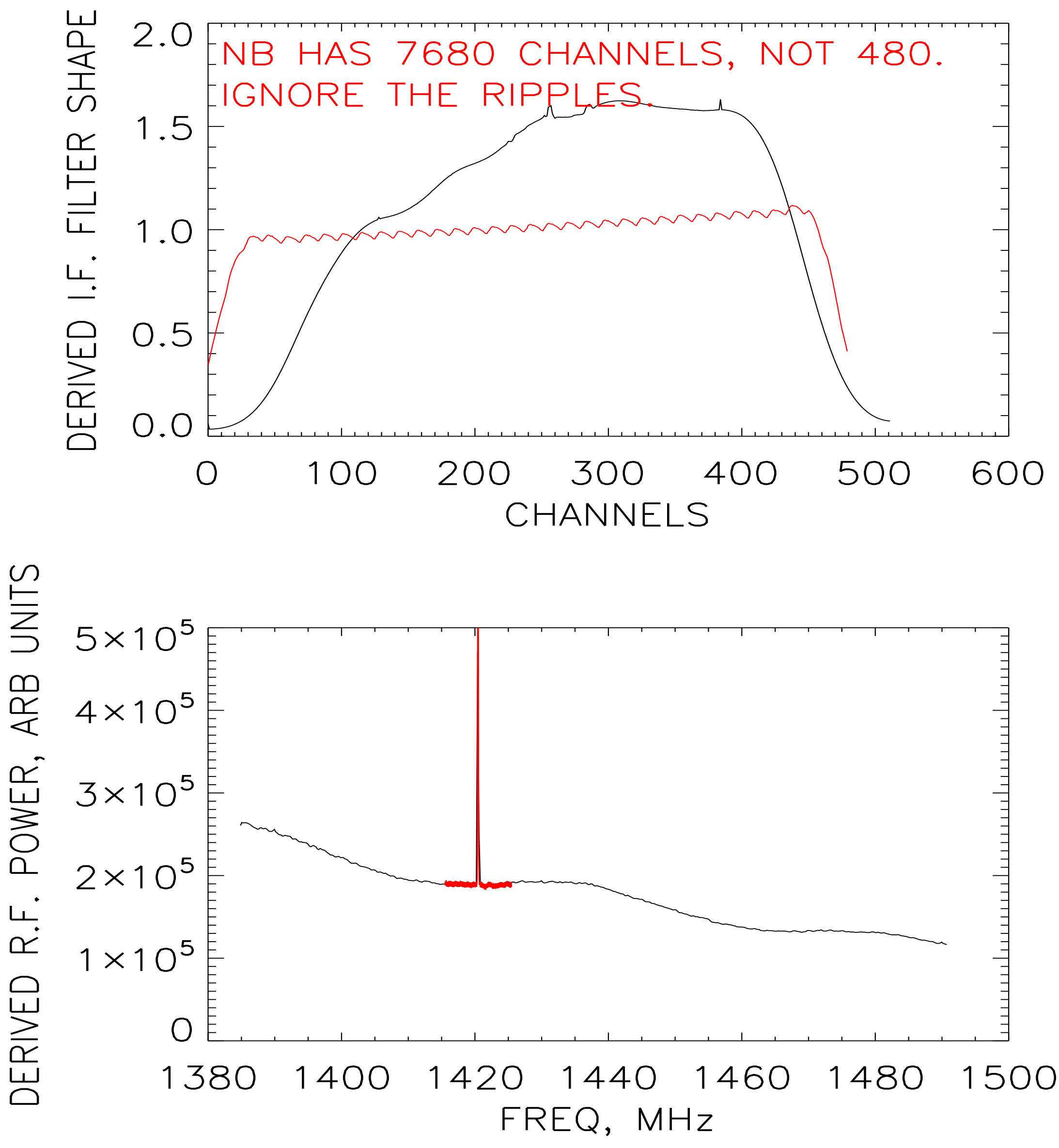
LEAST-SQUARES FREQUENCY SWITCHING: NEW!!

With GALFA, we avoid frequency switching and directly determine $G_{IF} f_{(IF)}$. We observe one position using seven local oscillator frequencies and use a least-squares analysis.


OBSERVED POWER, 7 L.O. FREQS
6×10^5
4×10^5
2×10^5
0
0
100
200
300
400
500
600
CALIBRATION BAND CHANNELS
OBSERVED POWER, 7 L.O. FREQS
1×10^7
8×10^6
6×10^6
4×10^6
2×10^6
0
0
2000
4000
6000
8000
SCIENCE BAND CHANNELS

LSFS: Derived spectra

We obtain the filter shape and the r.f. power spectrum:



THEORY:

$G_{IF}(f_{IF})$ **is stable and changes negligibly during an observing session. Going back to our original equation for** $M(f_{IF})$**, after dividing by** $G_{IF}(f_{IF})$ **we obtain**

$$M(f_{IF}) = G_{RF}(f_{RF}) \left\{ \underbrace{(T_*(f_{RF}) + T_R(f_{RF}))}_{spectral} + \underbrace{(T_* + T_R)}_{continuum} \right\}$$

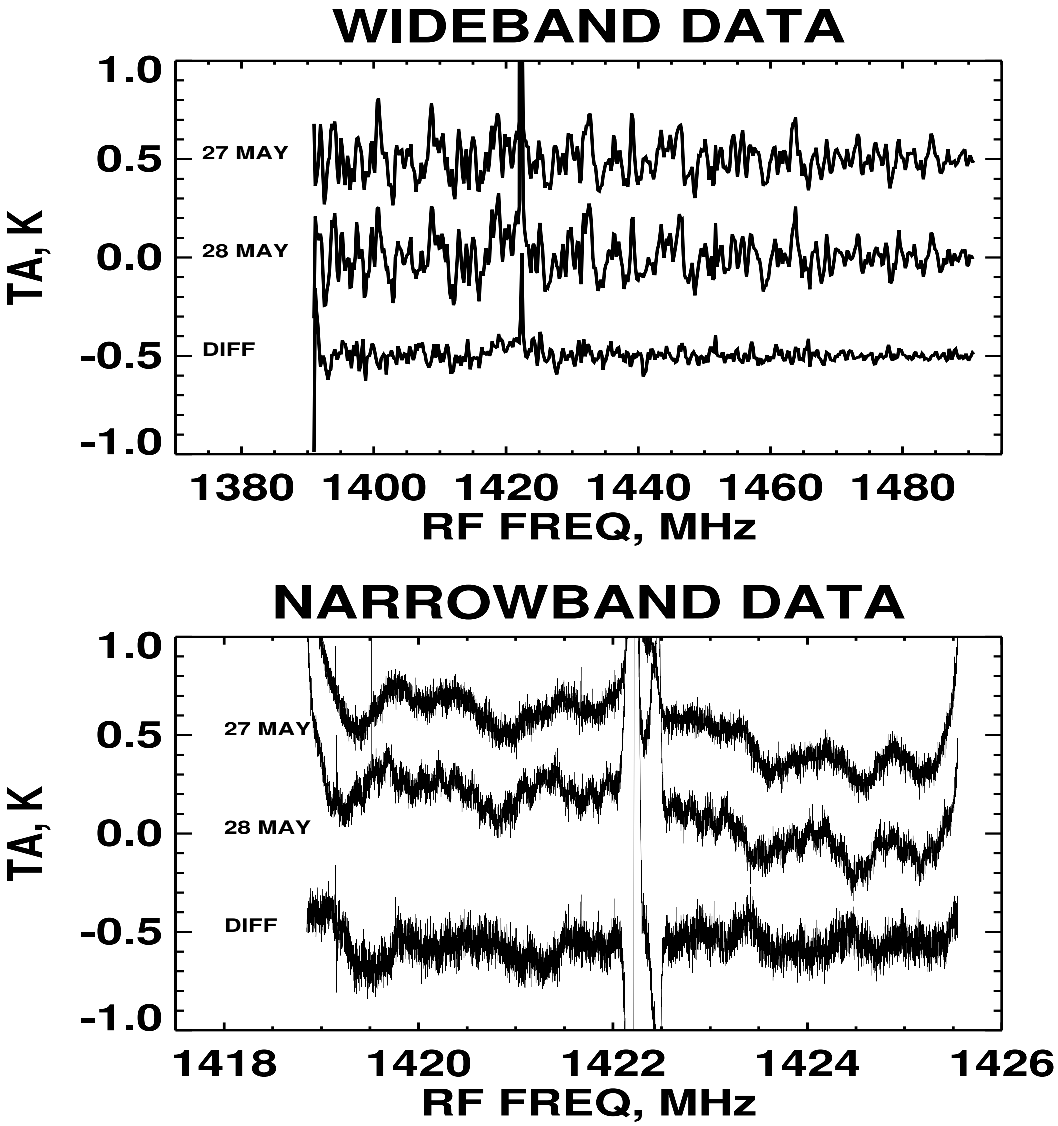
Now, one normally assumes:

- $G_{RF}(f_{RF})$ **changes slowly with frequency and can be fit by a low-order polynomial.**
- $T_R(f_{RF})$ **changes slowly with frequency and can be fit by a low-order polynomial.**

This provides the *important conclusion* **that we can fit polynomials to our wideband calibration spectrum and interpolate over the narrowband frequency interval. This should lead to nearly perfect baselines.**

PRACTICE: PROBLEM WITH ALFA!!

The Problem: **One or both of these assumptions is incorrect. We find fairly rapidly-varying** *pattern noise* **in our spectra. This repeats almost exactly from day to day. We are currently investigating this situation. . .**



WIDEBAND DATA
1.0
0.5
0.0
-0.5
-1.0
TA, K
27 MAY
28 MAY
DIFF
1380
1400
1420
1440
1460
1480
RF FREQ, MHz
NARROWBAND DATA
1.0
0.5
0.0
-0.5
-1.0
TA, K
27 MAY
28 MAY
DIFF
1418
1420
1422
1424
1426
RF FREQ, MHz