

Maximum Beam Separation for Multi Beam Nyquist Sampling with a Symmetric Elliptical Footprint Path

Germán Cortés Medellín
 NAIC, Cornell University
 Org. June 07, 2002
 Rev. June 12, 2002

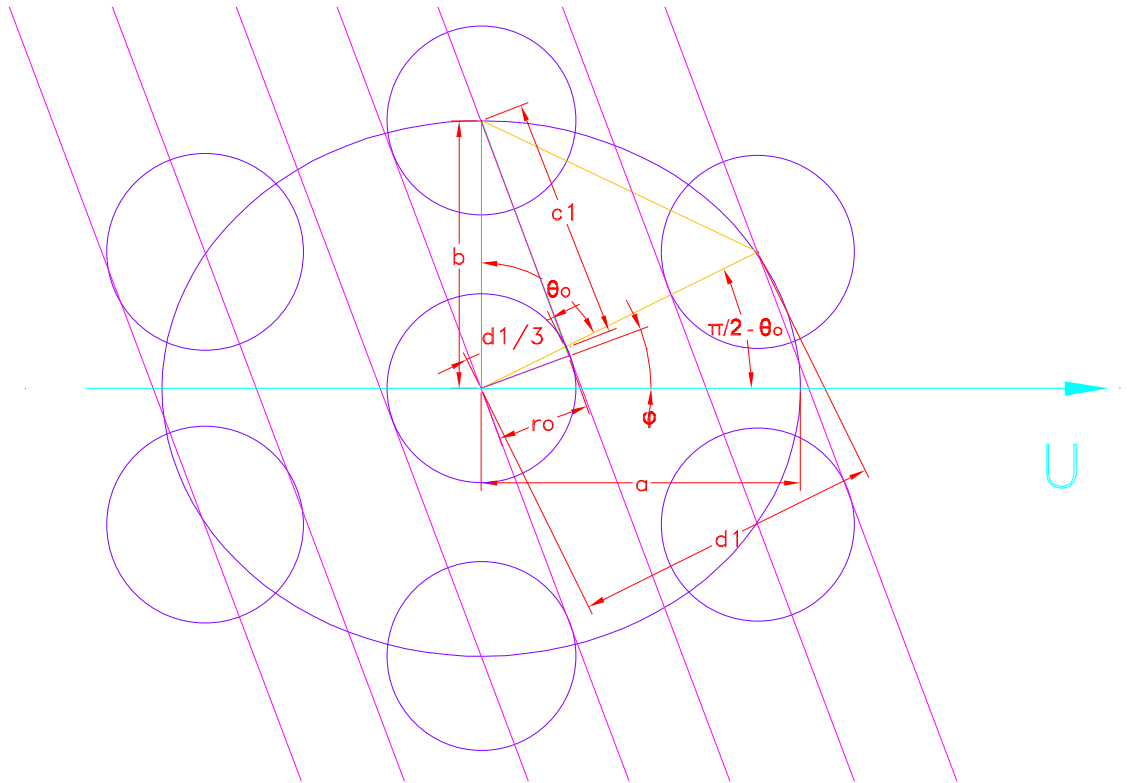


Figure 1. Geometry of the multi-beam 3 dB footprint in the sky

1 Introduction

Figure 1 shows the general geometry for the multi-beam 3 dB footprint in the sky for a given horn center-to-center separation of the Arecibo multi-beam system. For the configuration shown in Figure 1, we intent to answer two questions: first, what is the angle ϕ that we need to rotate the *feed arm*¹ in order to sample in a drift scan mode; second, what is the maximum value of b , the minor semi-axis of the elliptical path, for a given *beam radius* r_o , which allows nearly uniform drift scan with the peak beam tracks touching tangentially the adjacent beams.

2 Assumptions

We considered the following assumptions:

¹The multi-beam could also be rotated instead, nevertheless, the required calculation for this is more involved and requires further work compared with the simple calculation presented here.

- The beams adjacent to the central beam move on an elliptical path in the sky, with a eccentricity of ϵ , and major and minor semi-axis a and b respectively, as the multi beam rotates in the focal plane.
- The array is shifted along the X-axis in the focal plane to make more symmetric the scanning loses for feeds 1 to 6. This makes the central beam and the upper and lower beams in the figure to shift a few arc seconds off the center of the ellipse. We are ignoring this effect.
- The central Pixel has higher gain, and therefore, a larger beam size in the sky when referenced to the adjacent beams. We are ignoring this effect.
- In addition, the angle θ_o is slightly different for the right and left beams, in Figure 1. We are ignoring this and other any variations in beam size and location as a function of angle due to the characteristics of the FOV.
- The horizontal axis U in Figure 1 indicates the direction of the antenna feed arm.
- For a drift scan we rotate the feed arm an angle ϕ clockwise, that we are going to obtain here.
- At the Nyquist sampling the peak of a beam touches tangentially the 3 dB level of the next beam track.
- The 3 dB beam radius is defined as r_o .

3 Calculations

From the figure we obtain the first equation, namely,

$$b = \frac{r_o}{\cos[\theta_o + (\pi/2 - \theta_o) - \phi]} = \frac{r_o}{\sin \phi} \quad (1)$$

d_1 is obtained from the condition,

$$\begin{aligned} \frac{d_1^2 \cos^2(\pi/2 - \theta_o)}{a^2} + \frac{d_1^2 \sin^2(\pi/2 - \theta_o)}{b^2} &= 1 \\ \frac{d_1^2 \sin^2 \theta_o}{a^2} + \frac{d_1^2 \cos^2 \theta_o}{b^2} &= 1 \end{aligned} \quad (2)$$

Therefore,

$$d_1 = \frac{1}{\sqrt{\frac{\sin^2 \theta_o}{a^2} + \frac{\cos^2 \theta_o}{a^2(1-\epsilon^2)}}} = \frac{b}{\sqrt{1-\epsilon^2 \sin^2 \theta_o}} \quad (3)$$

As an intermediate step we calculate c_1 ,

$$\begin{aligned} c_1^2 &= b^2 + \frac{d_1^2}{9} - 2b \frac{d_1}{3} \cos \theta_o \\ c_1 &= b \sqrt{1 + \frac{1}{9(1-\epsilon^2 \sin^2 \theta_o)} - \frac{2 \cos \theta_o}{3 \sqrt{1-\epsilon^2 \sin^2 \theta_o}}} \end{aligned} \quad (4)$$

Then,

$$\frac{c_1}{\sin \theta_o} = \frac{b}{\sin(\pi - \theta_o - \phi)}$$

From this we obtain an equation for ϕ ,

$$\sin(\theta_o + \phi) = \frac{\sin \theta_o}{\sqrt{1 + \frac{1}{9(1 - \epsilon^2 \sin^2 \theta_o)} - \frac{2 \cos \theta_o}{3\sqrt{1 - \epsilon^2 \sin^2 \theta_o}}}} \quad (5)$$

Equation 5 gives the value of ϕ , whereas, from Equation 1 we obtain ellipse minor semi-axis b .

For the Multi-Beam, $\epsilon = 0.5444227$, is a weak function of the center-to-center horn separation. For 26 cm center-to-center distance, the value of $\theta_o = 64.6^\circ$ approximately. With these we obtain that $\phi = 22.48^\circ$. As a comparison, for the circular case, $\epsilon = 0$ and $\theta_o = \pi/3$, we obtain, $\phi = 19.11^\circ$.

Measured data for AO suggest that the typical 3 dB beam size for the L-Band is $186'' \times 210''$, which put $2b$ between $549''$ and $486''$. With a scale of $2b = 742''$ for a center-to-center separation of 26 cm, under this conditions these values put the maximum center-to-center separation between 17 and 19 cm.

4 Comments

- This solution is not unique. There are actually three other solutions for the configuration shown in Figure 1. Alternatively, four other solutions, occur when the multi beam is rotated by 90° (Not the feed arm!), and three beams in the sky are aligned along the U -direction.
- It should be emphasized that the actual beams are elliptical, and
- The gain of the central pixel is higher than the adjacent beams, making the beam size effectively larger at the equivalent sensitivity of the other beams².
- These effects make that the maximum value for b to be somewhat larger than Equation 1 would predict.
- In addition, rotating the multi-beam also helps to equalize tracks during a drift scan for a uniform sampling. In practice, Nyquist sampling in drift mode will be possible with a suitable combination of multi-beam rotation plus feed arm rotation.

²A more clear 3D picture of this may be found in the front page of *Nature*, Jan. 3, 2002, although the picture refers to ultra cold atoms entering a new phase, it illustrate the point of having a higher central pixel gain when defining the 3 dB level of adjacent beams