RECEIVER CALIBRATION WITH TWO TEMPERATURE LOADS

This memo summarizes fairly well–known results regarding the uncertainty in determining a receiver noise temperature when the two temperature loads available are both at much higher temperatures than the system temperature. The exigencies of ALFA calibration suggest we reexamine this issue. My conclusion is that there is little hope of getting a meaningful result, due to the inevitable uncertainties in the effective temperature of the loads. A reasonable estimate for these suggests that the uncertainty in the receiver temperature will be 5 K, or just about equal to the expect value.

1 Receiver Calibration

We assume that a calibration load has an effective temperature T defined such that the single–polarization power radiated per mode in the Rayleigh–Jeans limit is just kT. Then, if we have two loads with effective temperatures T_1 and T_2, and the system temperature is T_s, the total power output of the system will be given by

$$P_1 = \alpha[T_s + T_1] \ , \tag{1}$$

and

$$P_2 = \alpha[T_s + T_2] \ . \tag{2}$$

We have subsumed the power gain, G, and the measurement bandwidth $\delta\nu$ into α, given by

$$\alpha = Gk\delta\nu \ . \tag{3}$$

From the ratio of the two power outputs measured, we form the "Y–factor" defined by

$$Y = \frac{P_2}{P_1} \ , \tag{4}$$

and from the measured Y–factor we solve for the system temperature with the expression

$$T_s = \frac{T_2 - YT_1}{Y - 1} \ . \tag{5}$$

2 Errors or Uncertainties in Effective Temperature of Loads

The above procedure assumes that one knows the effective temperatures of the calibration loads. This is true only to some degree of precision. The actual values of the two temperatures are unimportant as long as we know them perfectly (assuming the system is linear, which could be an issue but which we will not deal with here). However, there will inevitably be some uncertainty in the effective temperatures, due to reflections, emissivity, absorption due to condensation or to atmospheric absorption. The last of these is not much of a problem at cm wavelengths, but is a big issue in the submm range. However, all the other problems do enter for cm wavelength calibrations, inasmuch as we are thinking in particular of free–space loads of eccosorb. Although the eccosorb itself may be at a well known temperature (ambient, or that of liquid nitrogen), the effective temperature may be different due to emissivity less than 1, and absorption in a layer of liquid water which may have condensed on the window one is looking through (for liquid nitrogen temperature load).

We thus consider the case where the **assumed** load temperatures differ from the real ones. The assumed temperatures are given by

$$T_1' = T_1 + \delta T_1 \ , \tag{6}$$

and

$$T_2' = T_2 + \delta T_2 \ . \tag{7}$$

To obtain the system temperature, we substitute these values in equation 5, but with the measured value of Y, which itself depends on T_1 and T_2. After some algebra, this gives us

$$T_s' = \frac{T_2'(T_s + T_1) - T_1'(T_s + T_2)}{T_2 - T_1} \ . \tag{8}$$

Defining the **error** in the system temperature to be

$$\delta T_s = T_s' - T_s \tag{9}$$

yields the expression for the error in the system temperature

$$\delta T_s = T_s \left(\frac{\delta T_2 - \delta T_1}{T_2 - T_1} \right) + \frac{T_1 \delta T_2 - T_2 \delta T_1}{T_2 - T_1} \ . \tag{10}$$

Note that we have not combined the uncertainties in a statiscal sense here. There is a clear, worst case situation, which is that the error in the system temperature is maximum positive if δT_2 is positive and δT_1 is negative (for

$T_2 > T_1$), while the error in the system temperature is maximum negative if δT_2 is negative and δT_1 is positive.

The first term is a multiplicative one, and assuming that we are in the limit where $T_s \ll T_1, T_2$, this term will be relatively insignificant compared to the second term, which is error of fixed magnitude.

3 Numerical Examples

For all of the following we assume a value $T_s = 5$.

[1] The first example is a reasonable guess for ambient temperature and liquid nitrogen calibration loads. $T_2 = 295 \;\; T_1 = 80; \;\; \delta T_2 = 5 \;\; \delta T_1 = -3$.

We find

$$\delta T_s = 0.0372 T_s + 5.977 = 6.16 \; . \tag{11}$$

Taking the other sign, we find $\delta T_s = -6.16$. The derived value of the system temperature would thus be 5 ± 6.16, which is not a reassuringly small uncertainty.

[2] The second example is using "cold sky" as a load, per standard JPL and Arecibo practice. There is uncertainty due to the sky temperature produced by atmospheric absorption, and due to spillover from far sidelobes (ground pickup). Note that both of these increase T_1. Typically, we make a calculation of the sky temperature based on standard curves, and may add a few degrees for ground pickup. If we take a reasonably generous value for the uncertainties, we might have $T_2 = 295 \;\; T_1 = 6; \;\; \delta T_2 = 5 \;\; \delta T_1 = -3$. This gives us

$$\delta T_s = 0.0277 T_s + 3.17 \; . \tag{12}$$

We see that the uncertainty half that above, but still is not impressively small. Even with the sky as a "cold" load, getting a really accurate measurement of T_s requires having the fractional precision of T_1 be reasonably good. For example, if we could reduce δT_1 to -1, we would obtain

$$\delta T_s = 0.0208 T_s + 1.125 = 1.22 \; . \tag{13}$$

This is not quite a 20 percent accuracy measurement of T_s, but at least would be good enough to say that a typical specification had been met, with reasonable confidence.

4 Conclusion

The error in derived system temperature depends both on the values of the warm and cold loads used, and their uncertainties. Reasonable uncertainties in the case of liquid nitrogen and ambient temperature loads make this measurement unlikely to give better than a 6 K uncertainty in measurement of a 5 K system temperature. A cold sky known to 1 K accuracy can give a 20 percent accurate measurement of the system temperature.